Nuclear Structure Of Neutron-Rich Nuclei In The Fission-Product Region: A Theoretical Perspective

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Many of the topics I discuss are treated in more detail in the more recent papers on my web page:

http://t16web.lanl.gov/Moller/abstracts.html

Nuclear Binding Energy BW (1935)

The nuclear binding energy according to BW is given by

$$B(N,Z) =$$

$$+a_{\mathrm{v}}A \qquad \text{(Volume energy)}$$

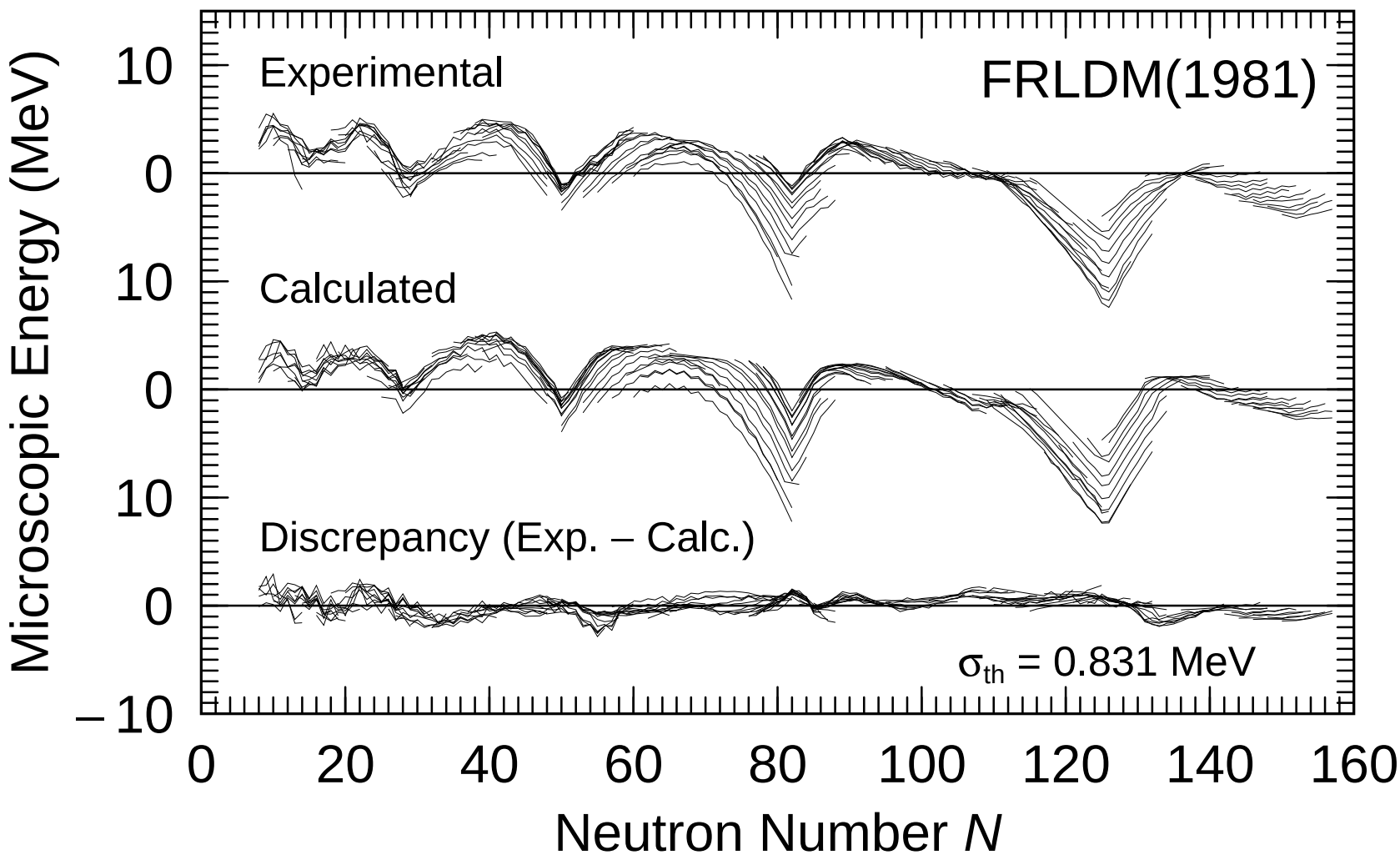
$$-a_{\mathrm{s}}A^{2/3} \qquad \text{(Surface energy)}$$

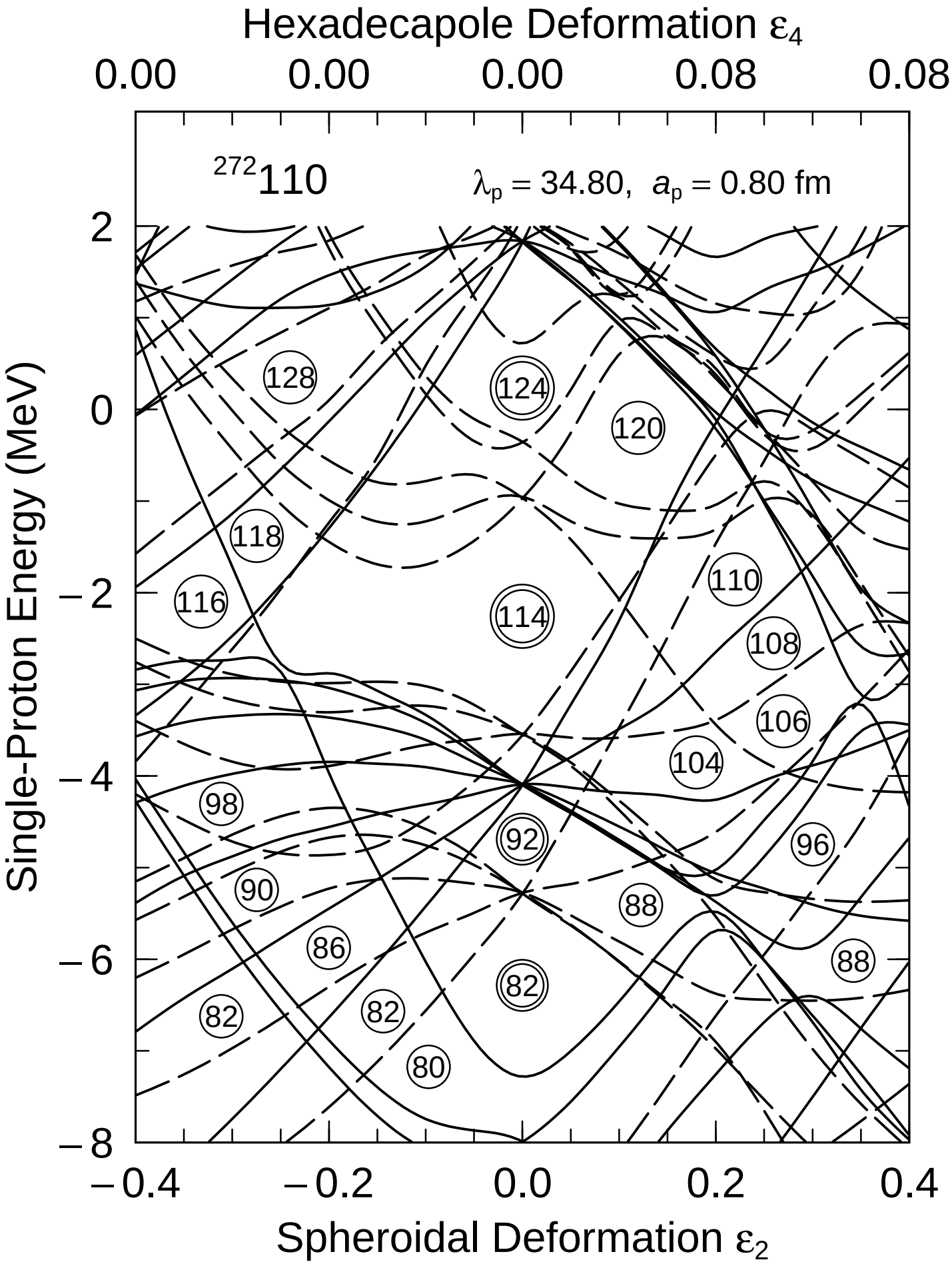
$$-a_{\mathrm{C}}\frac{Z^2}{A^{1/3}} \qquad \text{(Coulomb energy)}$$

$$-a_I\frac{(N-Z)^2}{A} \qquad \text{(Symmetry energy)}$$

$$-\delta(A) \qquad \text{(Pairing energy)}$$

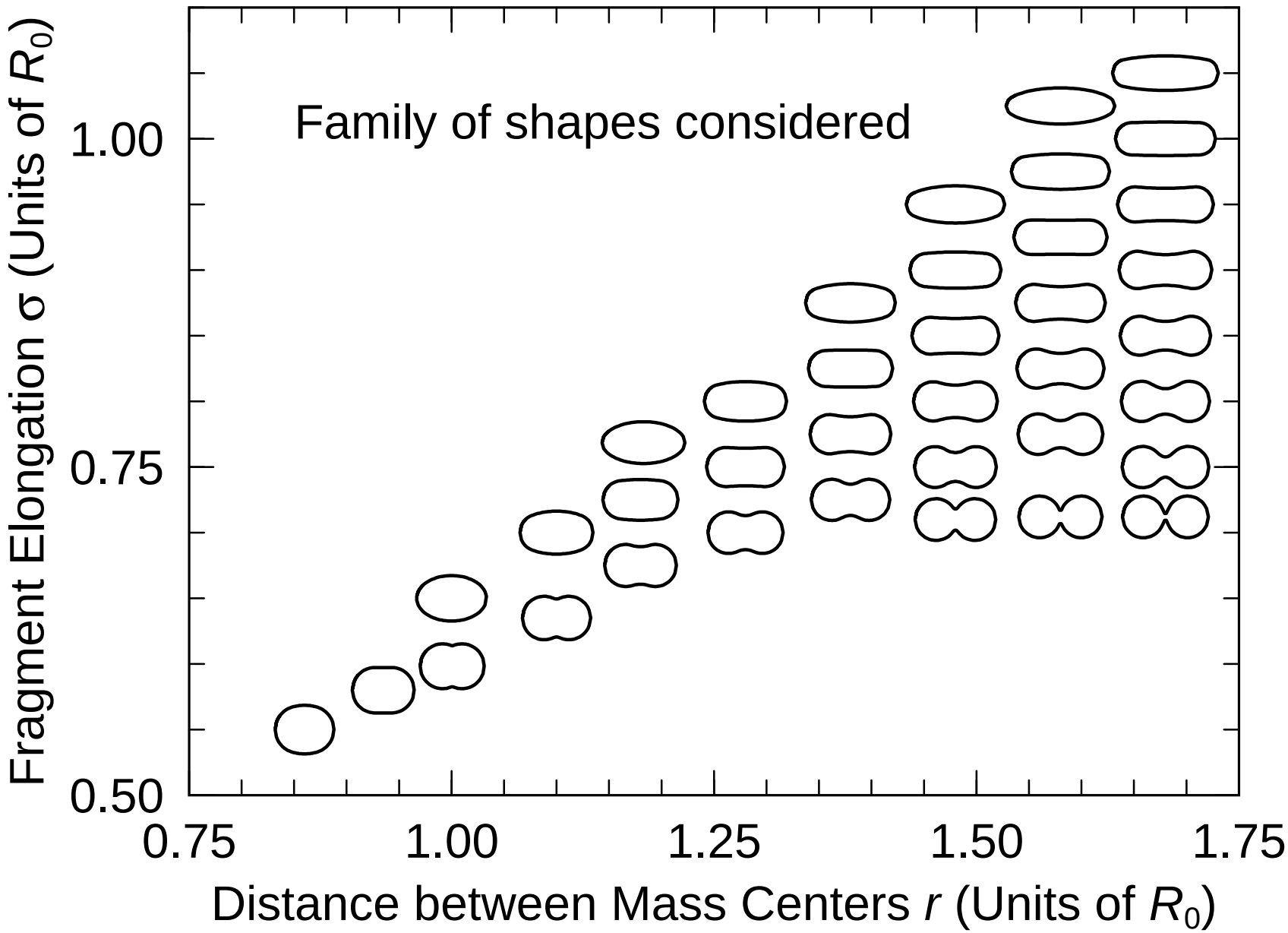
From: Nucl. Phys. A361 (1981) 117



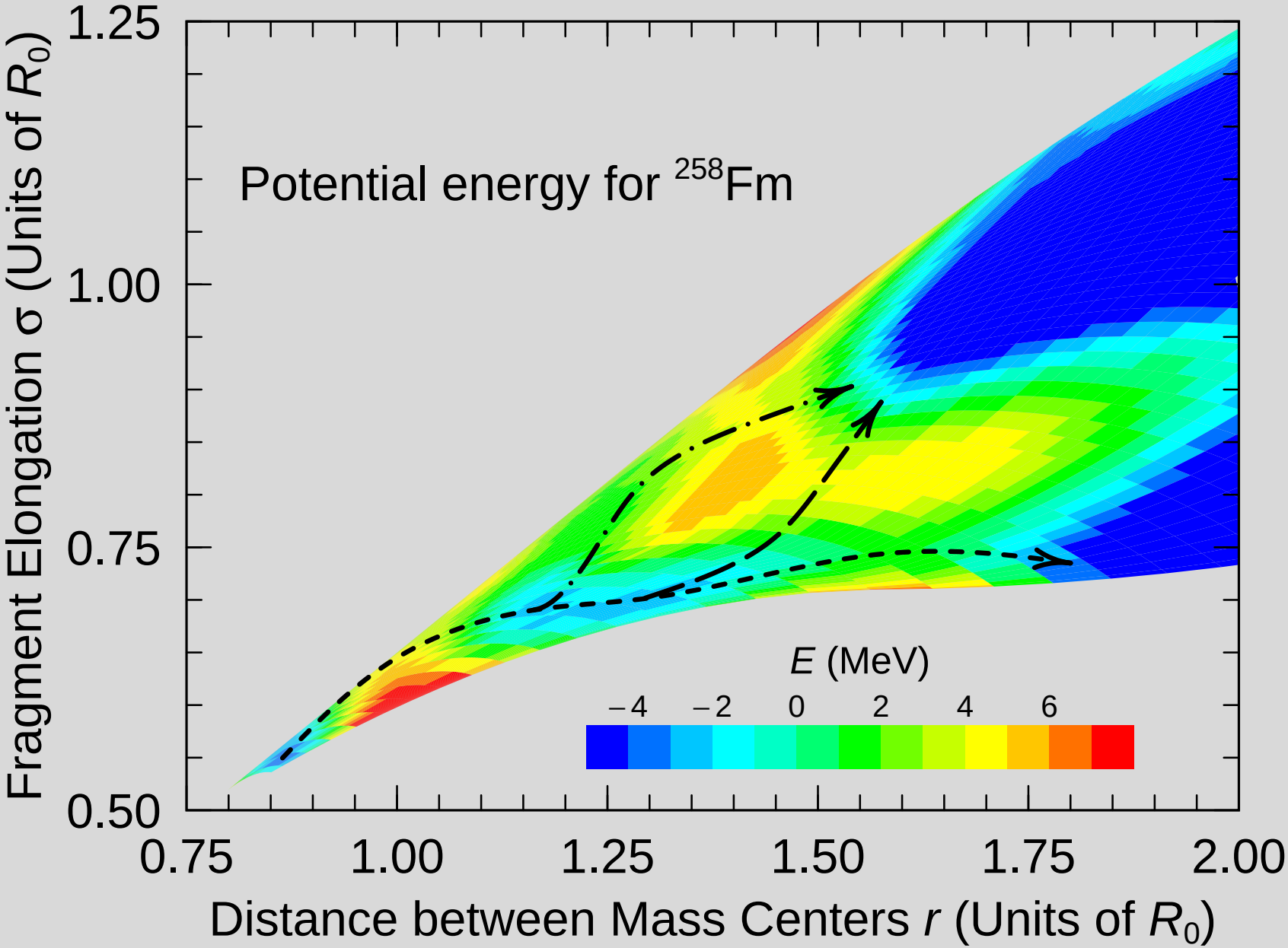


From: At. Data Nucl. Data Tables 66 (1997) 131

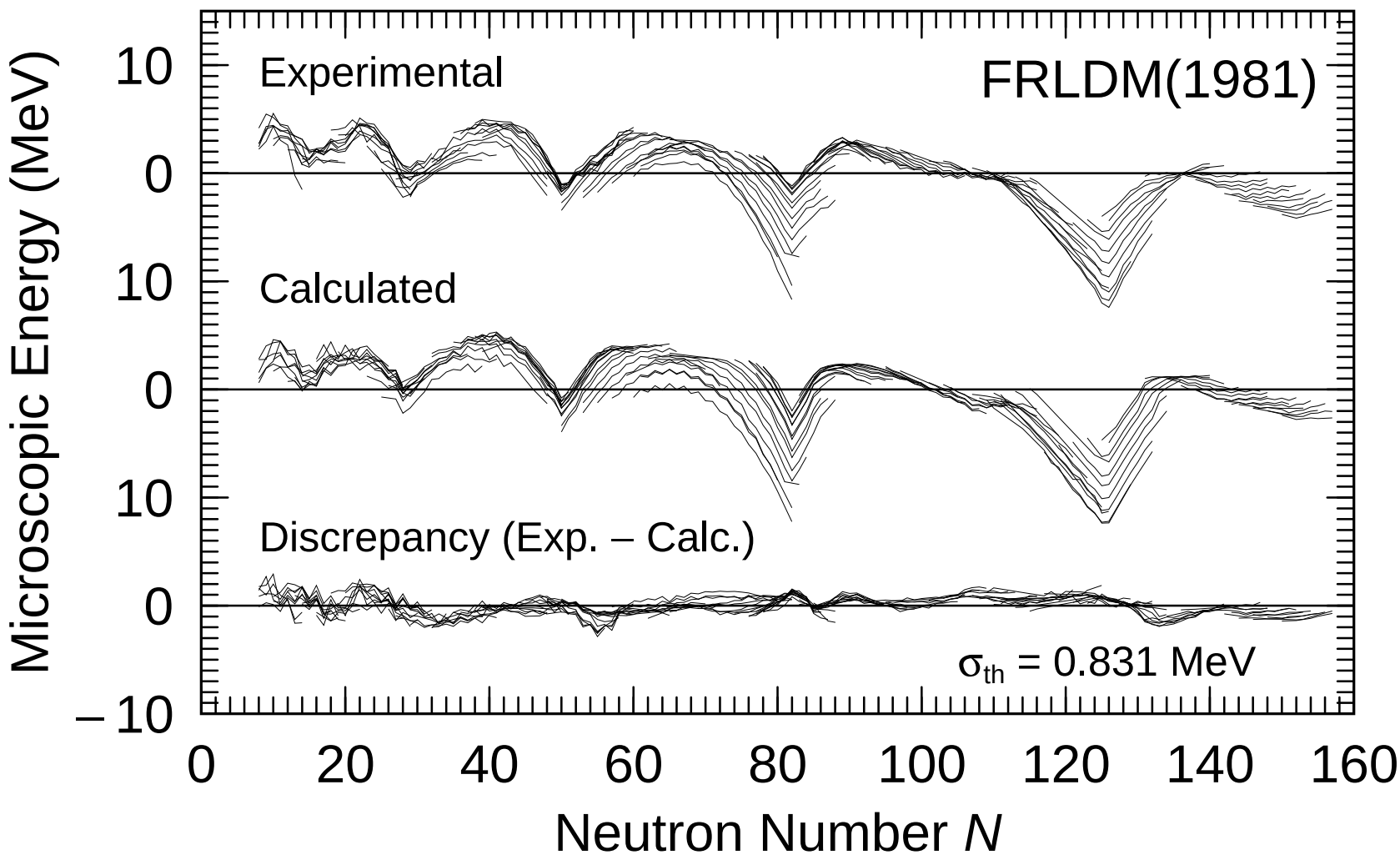
From: Nucl. Phys A469 (1987) 1



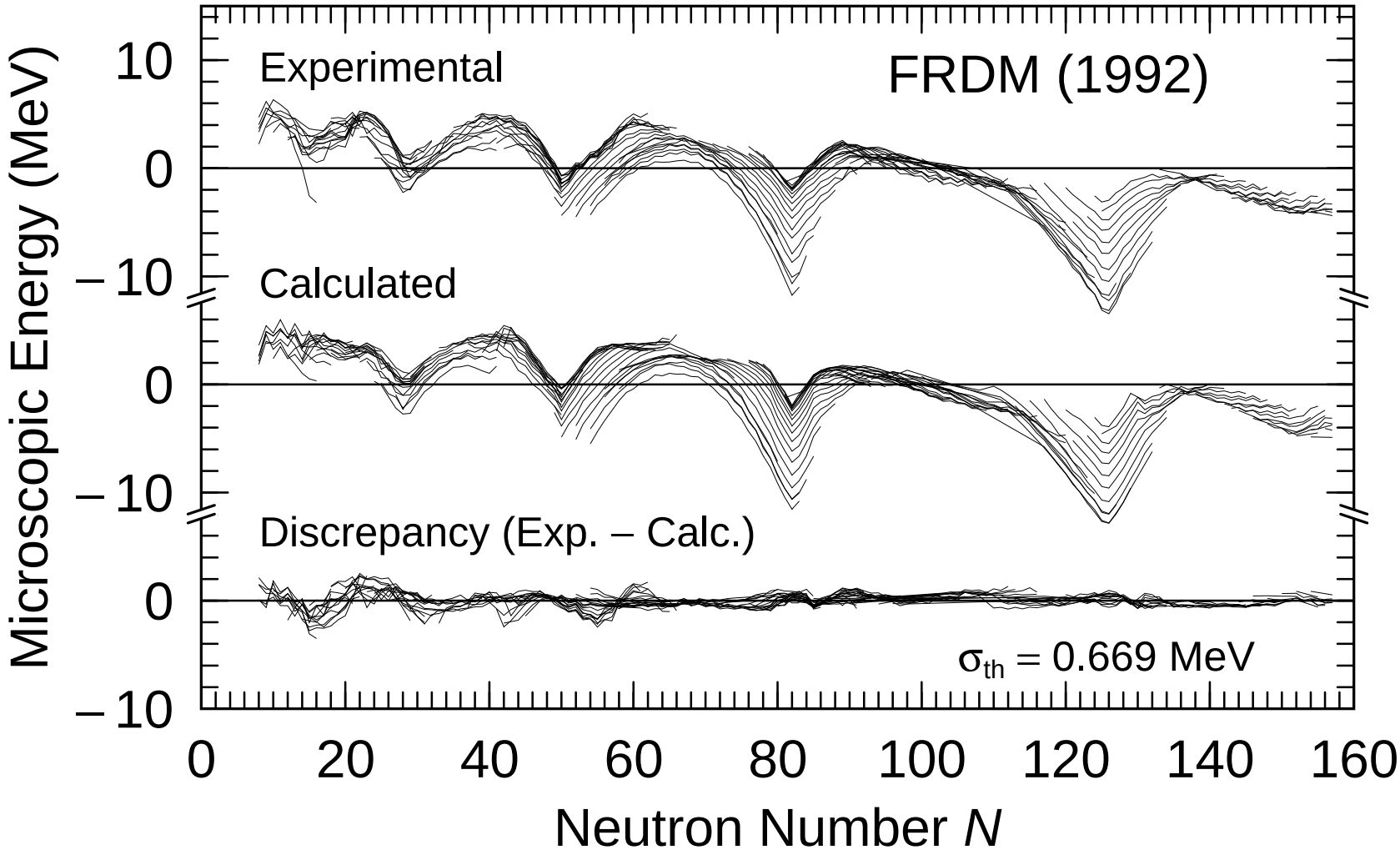
From: Journ. Phys. G: Nucl. Part. Phys. 20 (1994) 1681

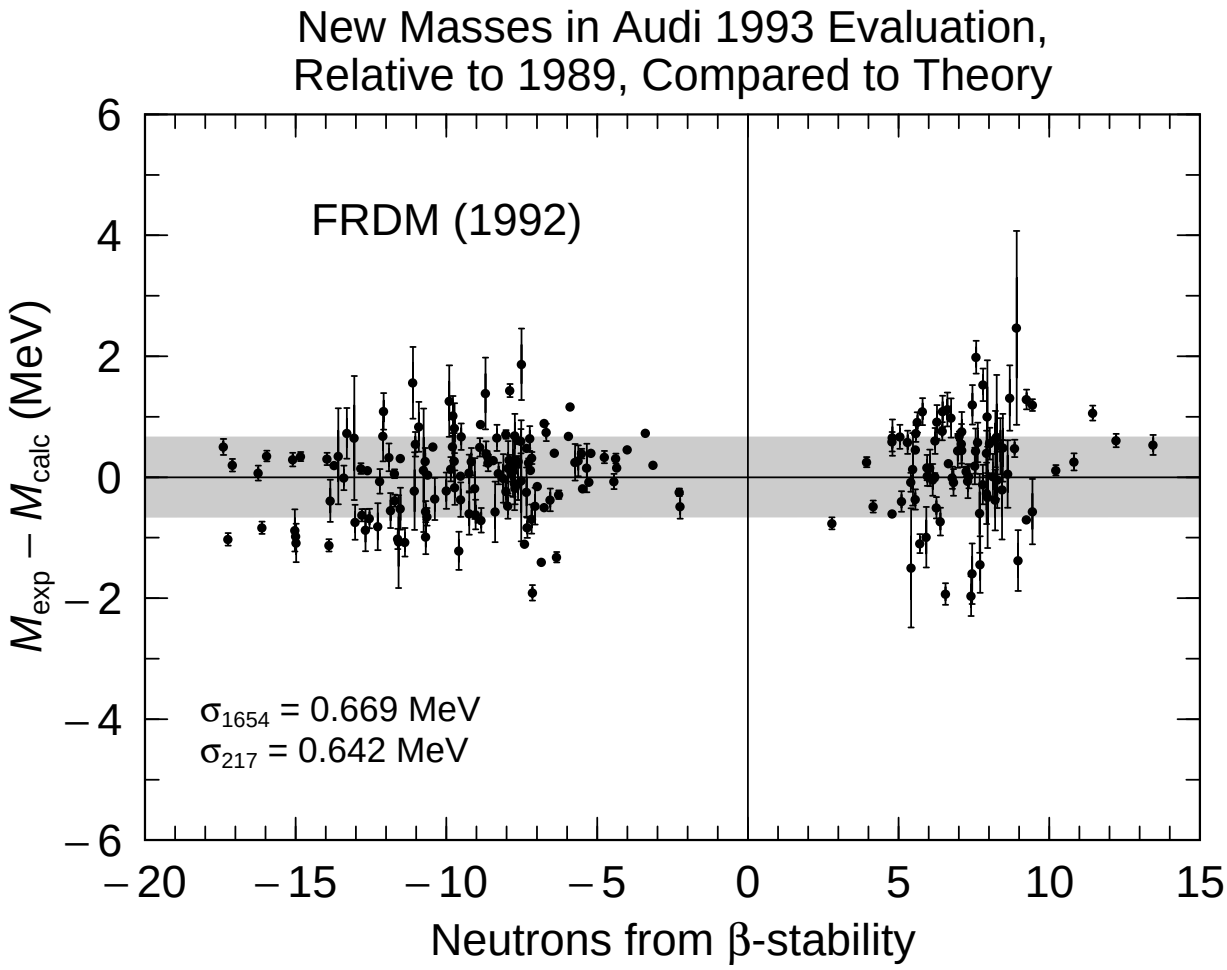


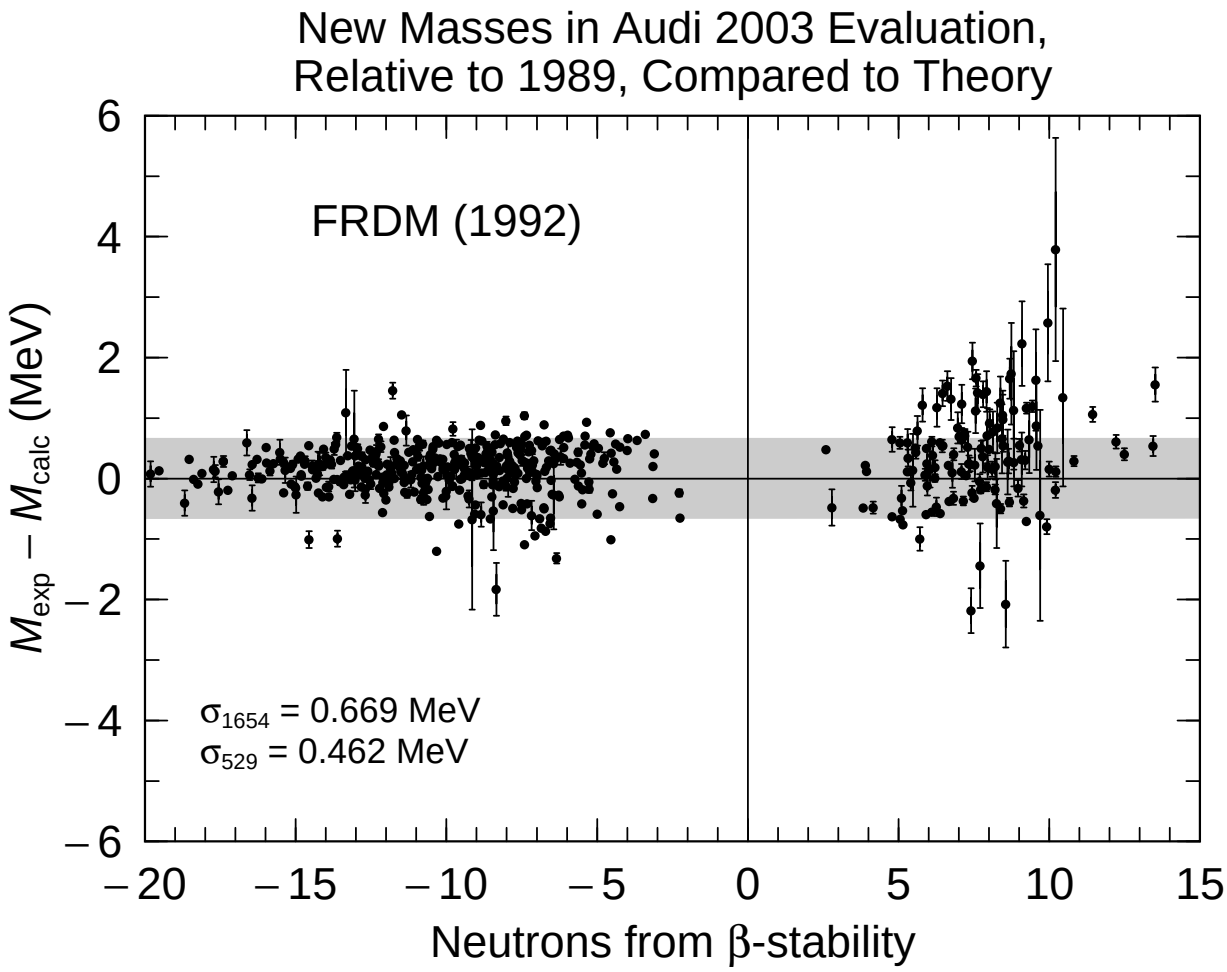
From: Nucl. Phys. A361 (1981) 117



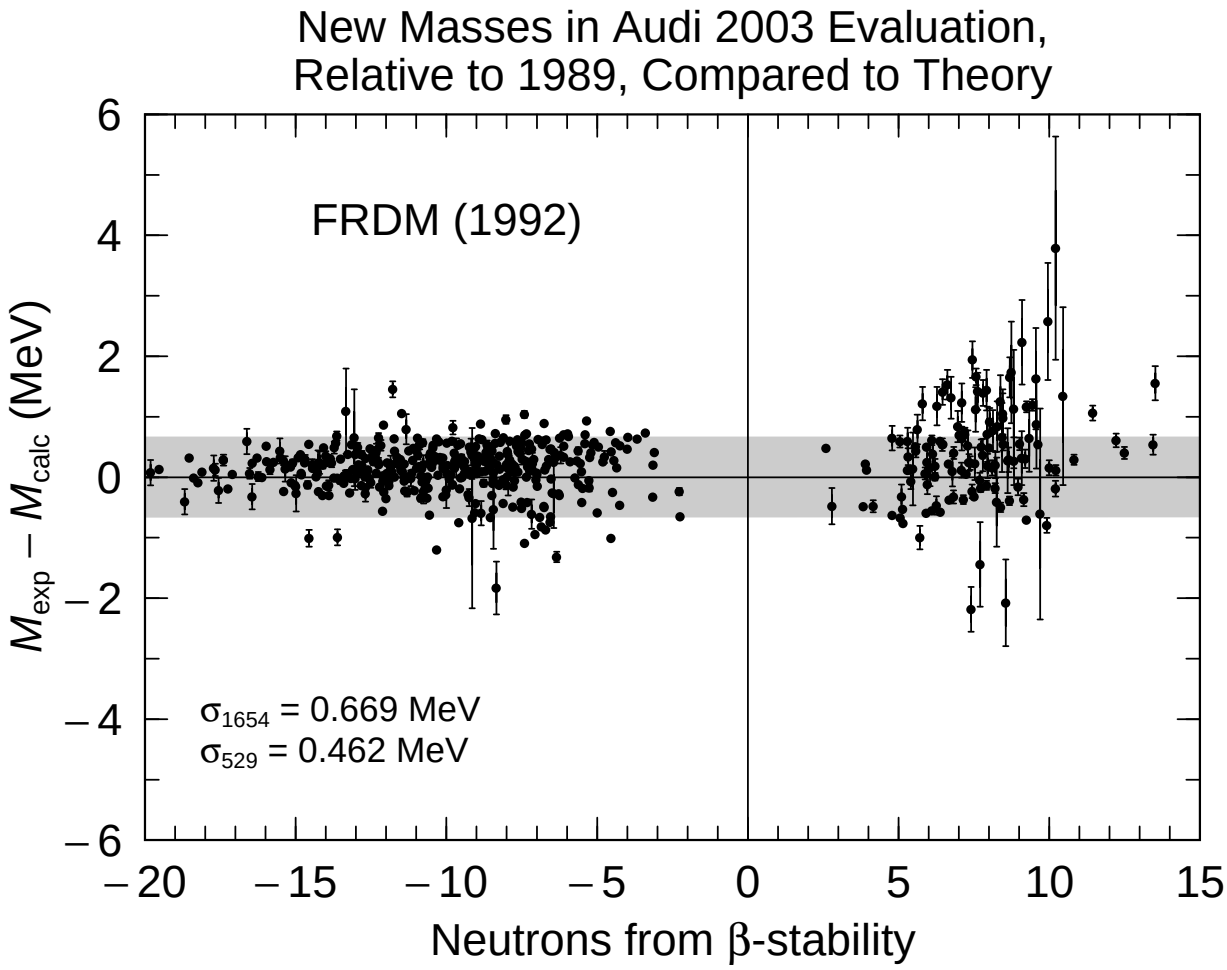
From: At. Data Nucl. Data Tables 59 (1995) 183

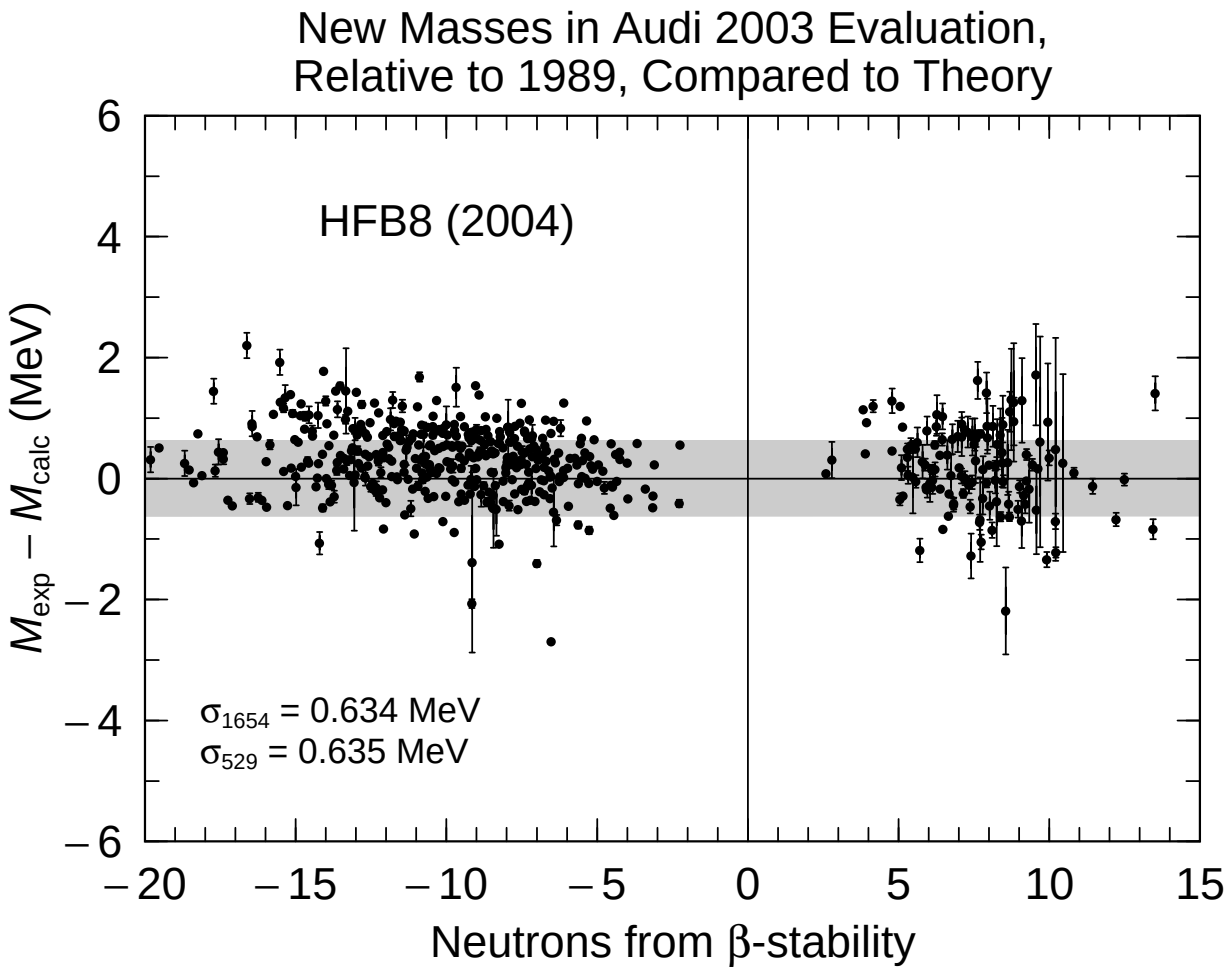






From: Proc. Int. Conf. on Nuclear Data and Technology
April 22-27, 2007, Nice, France, (EDP Sciences, (2008) p. 69)

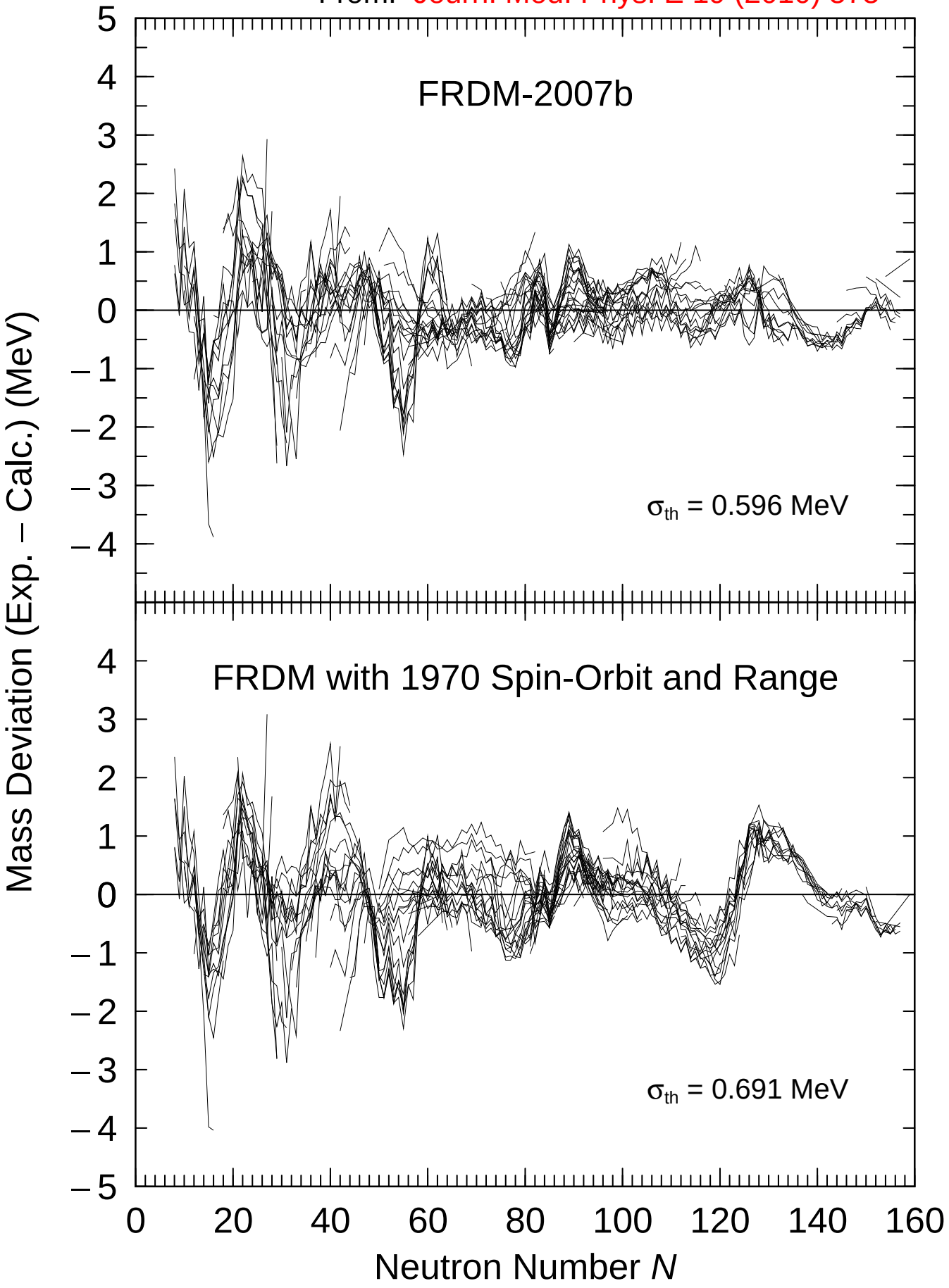




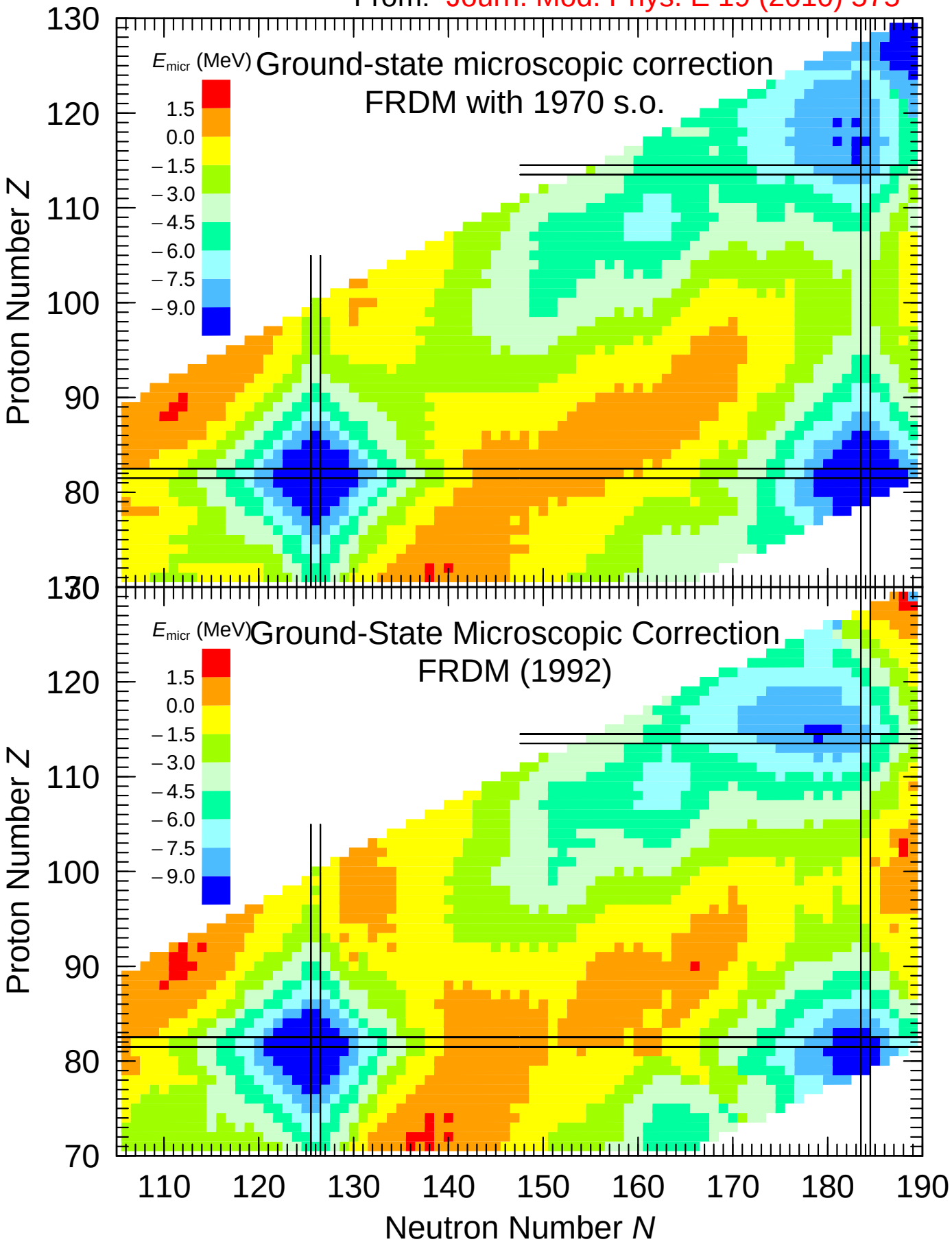
From: Proc. Int. Conf. on Nuclear Data and Technology
April 22-27, 2007, Nice, France, (EDP Sciences, (2008) p. 69)

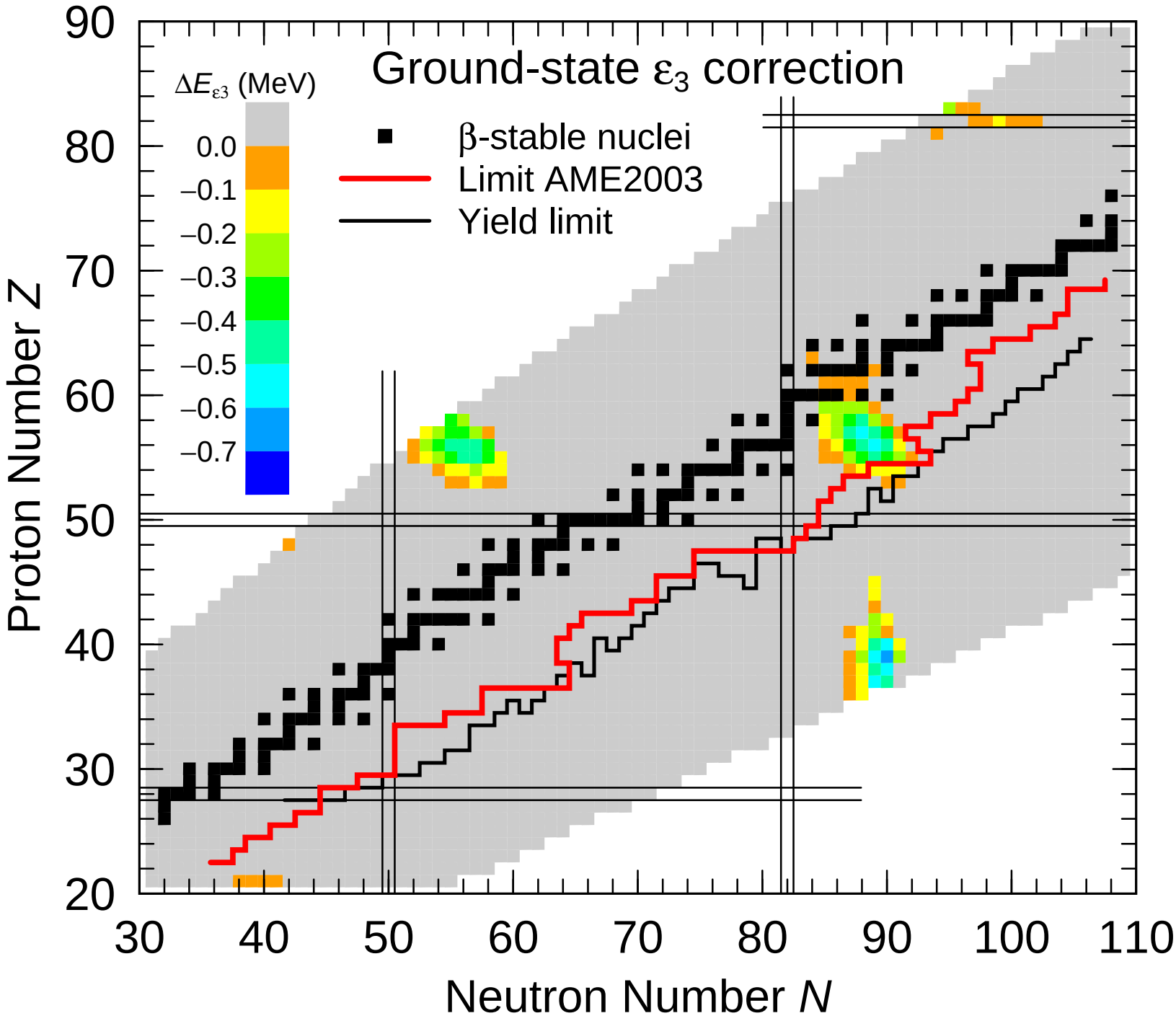
Model	A/C	a_1 (MeV)	a_2 (MeV)	J (MeV)	Q (MeV)	L (MeV)	$\mu_{\rm th}$ (MeV)	$\sigma_{{\rm th};\mu=0}$ (MeV)
(92)	1/1	16.247	22.92	32.73	29.21	0.00	0.0156	0.6688
(92)	1/3						0.1755	0.4617
(92)	1/2						0.0607	0.6314
(92)-a	1/1	16.245	23.02	32.22	30.73	0.00	0.0000	0.6614
(92)-a	1/3						0.0174	0.4208
(92)-a	1/2						0.0114	0.6180
(92)-b	1/1	16.286	23.37	32.34	30.51	0.00	0.0000	0.6591
(92)-b	1/3						0.0031	0.4174
(92)-b	1/2						0.0076	0.6157
(06)-a	2/2	16.274	23.27	32.19	30.64	0.00	0.0000	0.6140
(07)-b	2/2	16.231	22.96	32.11	30.83	0.00	0.0000	0.5964
(09)-a	1/1	16.143	22.41	33.00	27.50	86.22	0.0000	0.6166
(09)-a	1/3						0.0533	0.4091
(09)-a	1/2						0.0230	0.5808
(09)-b	2/2	16.145	22.43	32.53	28.53	72.59	0.0000	0.5788

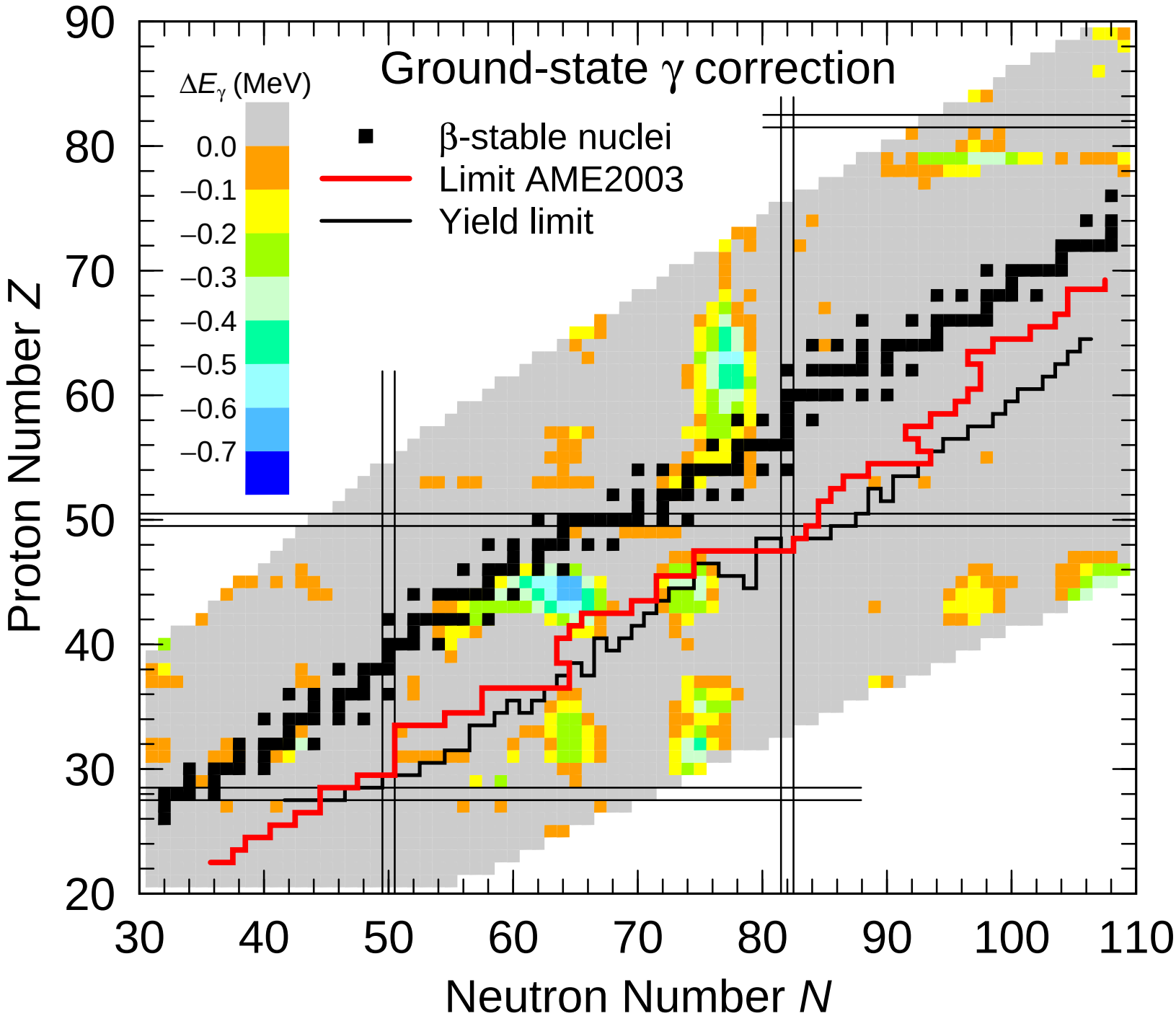
From: Journ. Mod. Phys. E 19 (2010) 575



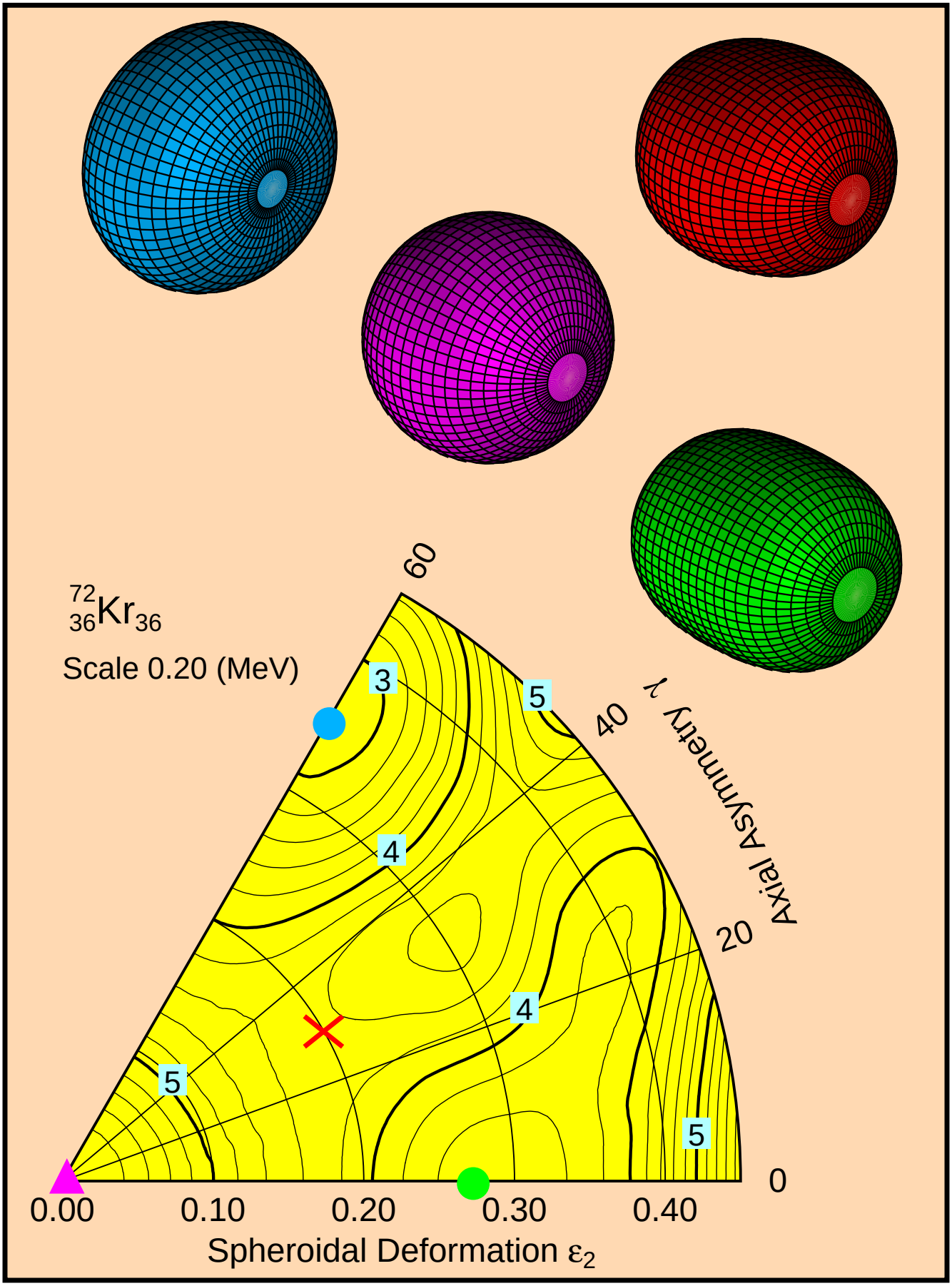
From: Journ. Mod. Phys. E 19 (2010) 575

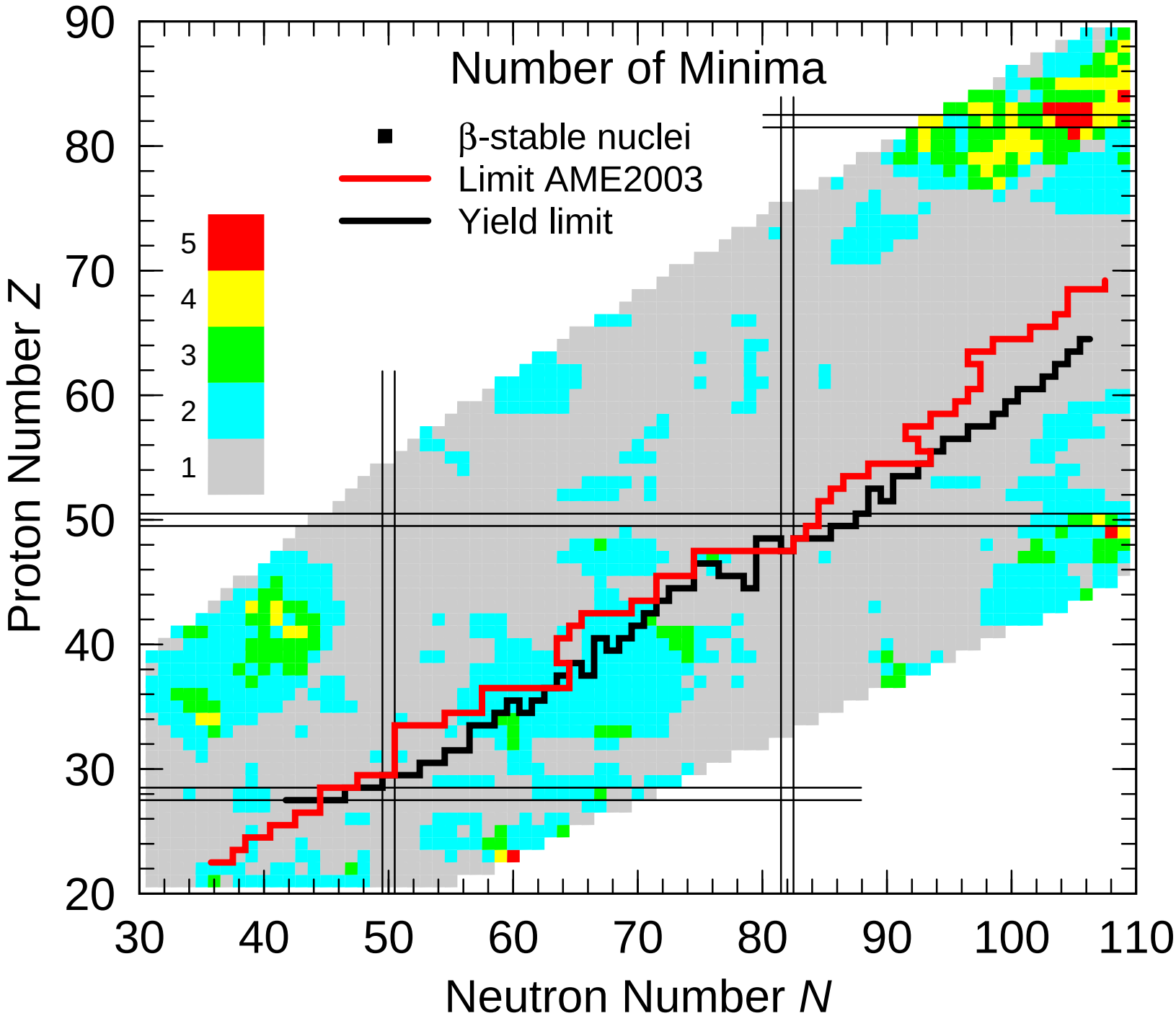


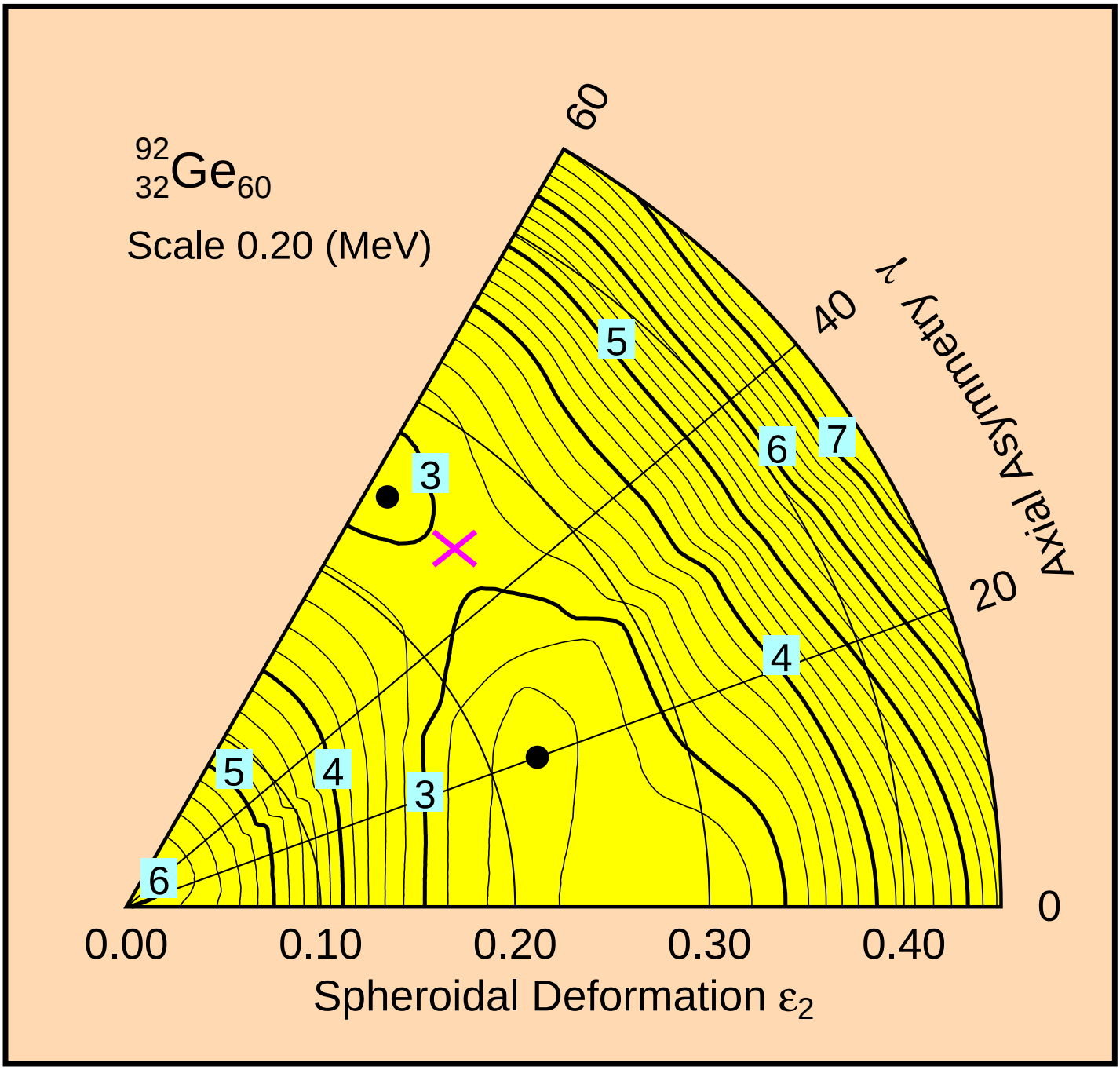

Ground-state ε3 correction
ΔEε3 (MeV)
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
β-stable nuclei
Limit AME2003
Yield limit
Proton Number Z
Neutron Number N
20
30
40
50
60
70
80
90
30
40
50
60
70
80
90
100
110

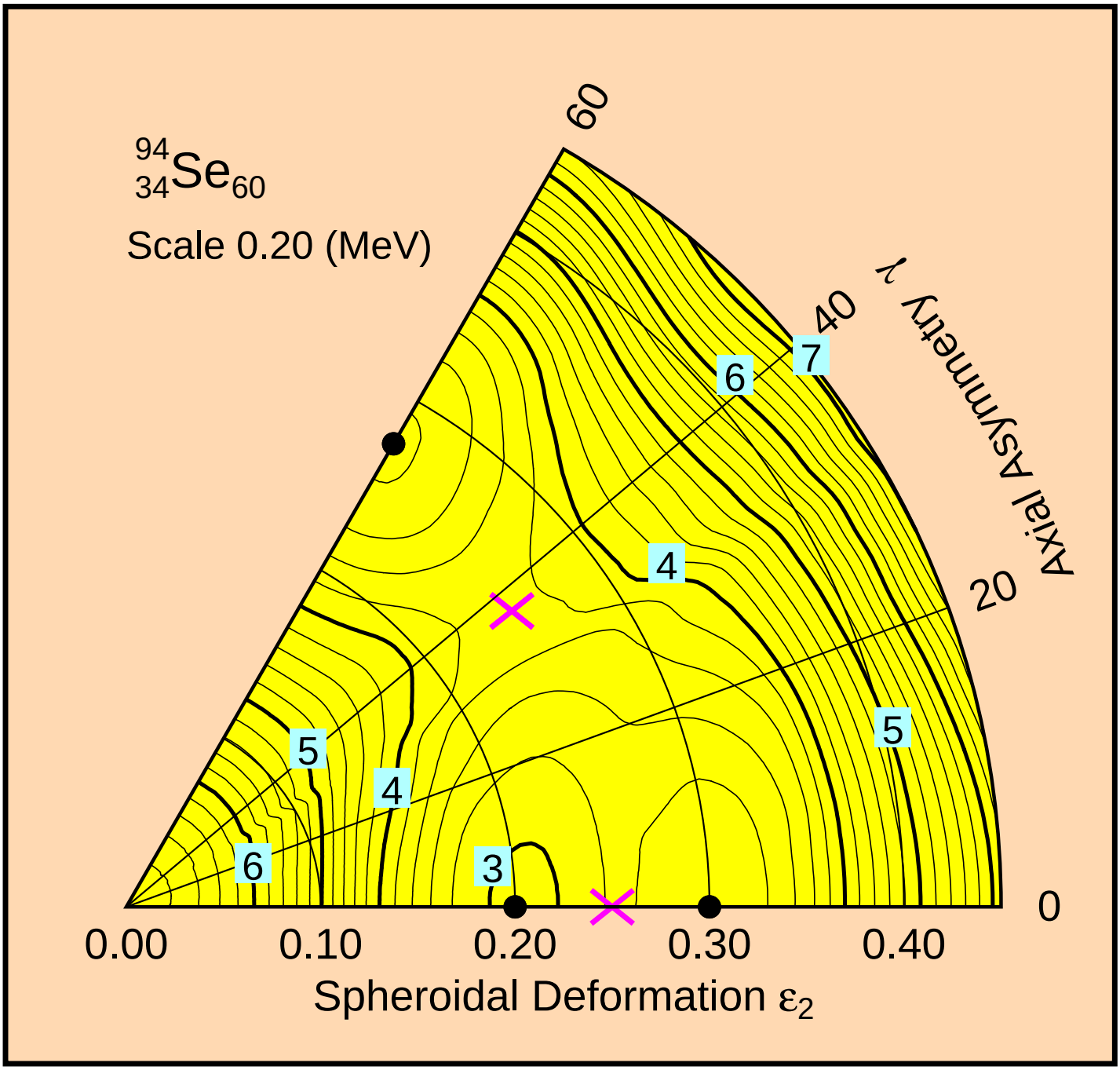

Ground-state γ correction
ΔEγ (MeV)
0.0
−0.1
−0.2
−0.3
−0.4
−0.5
−0.6
−0.7
β-stable nuclei
Limit AME2003
Yield limit
Proton Number Z
Neutron Number N
90
80
70
60
50
40
30
20
30
40
50
60
70
80
90
100
110

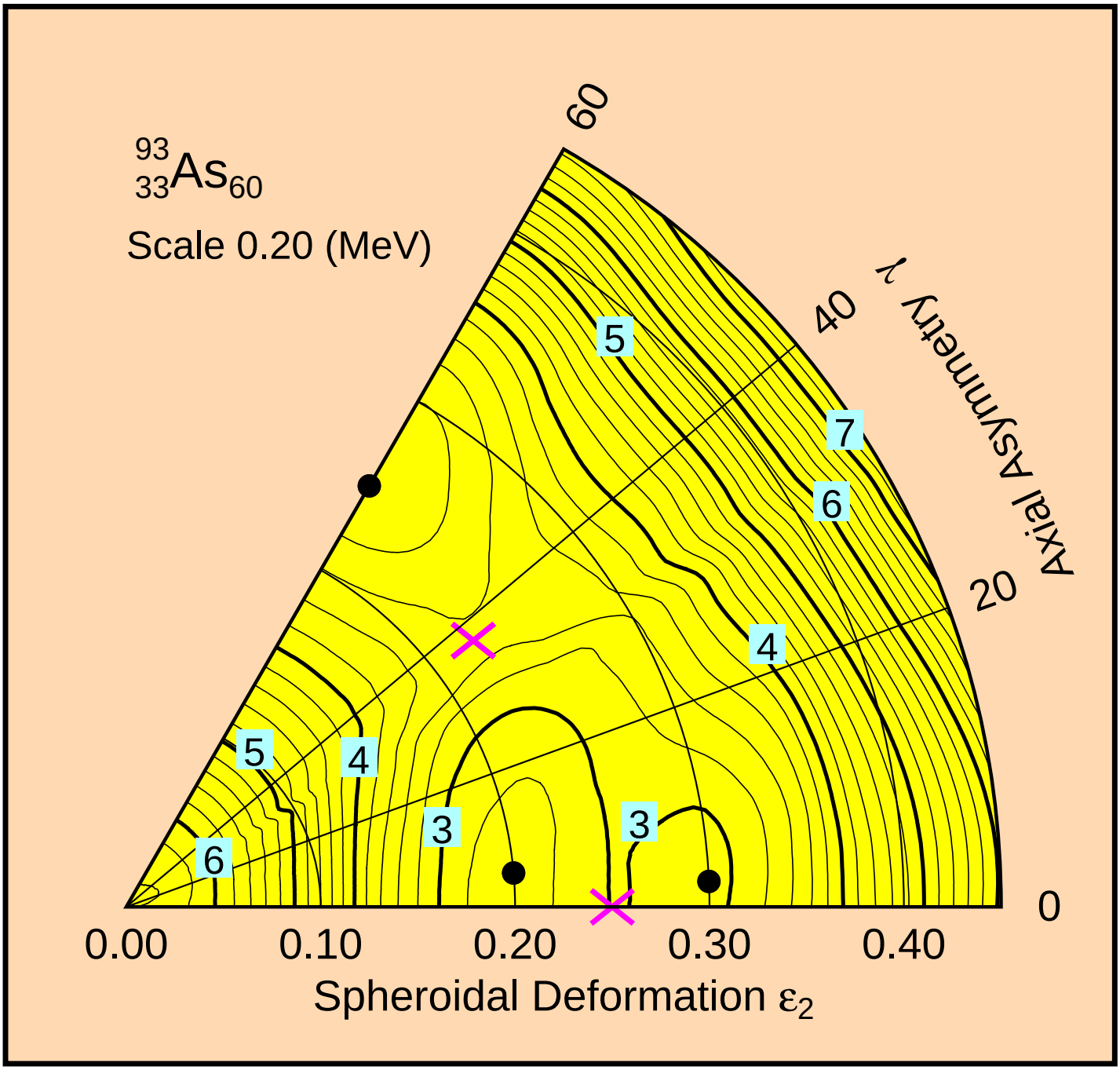
From: Phys. Rev. Lett. 103 (2009) 212501

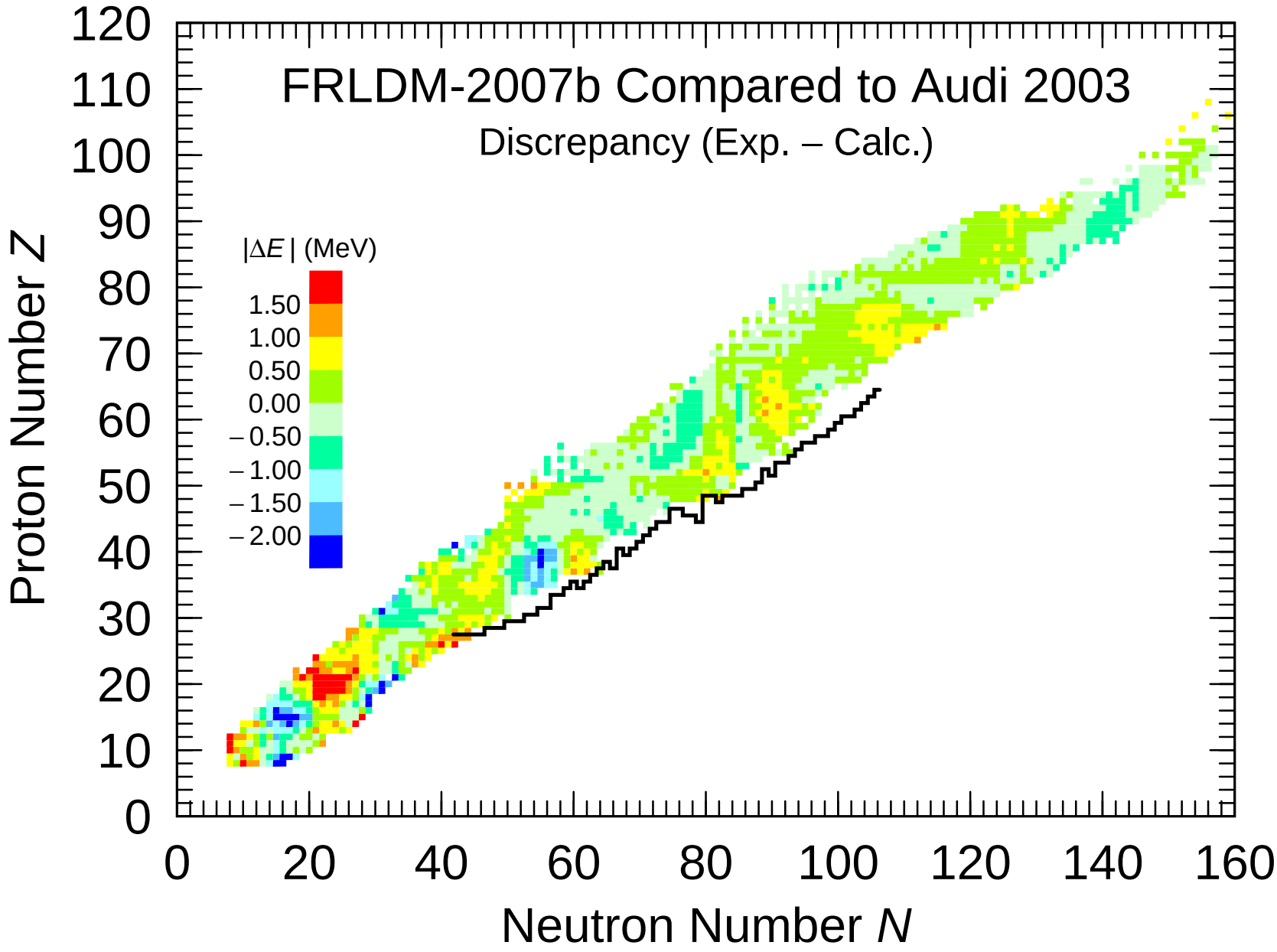


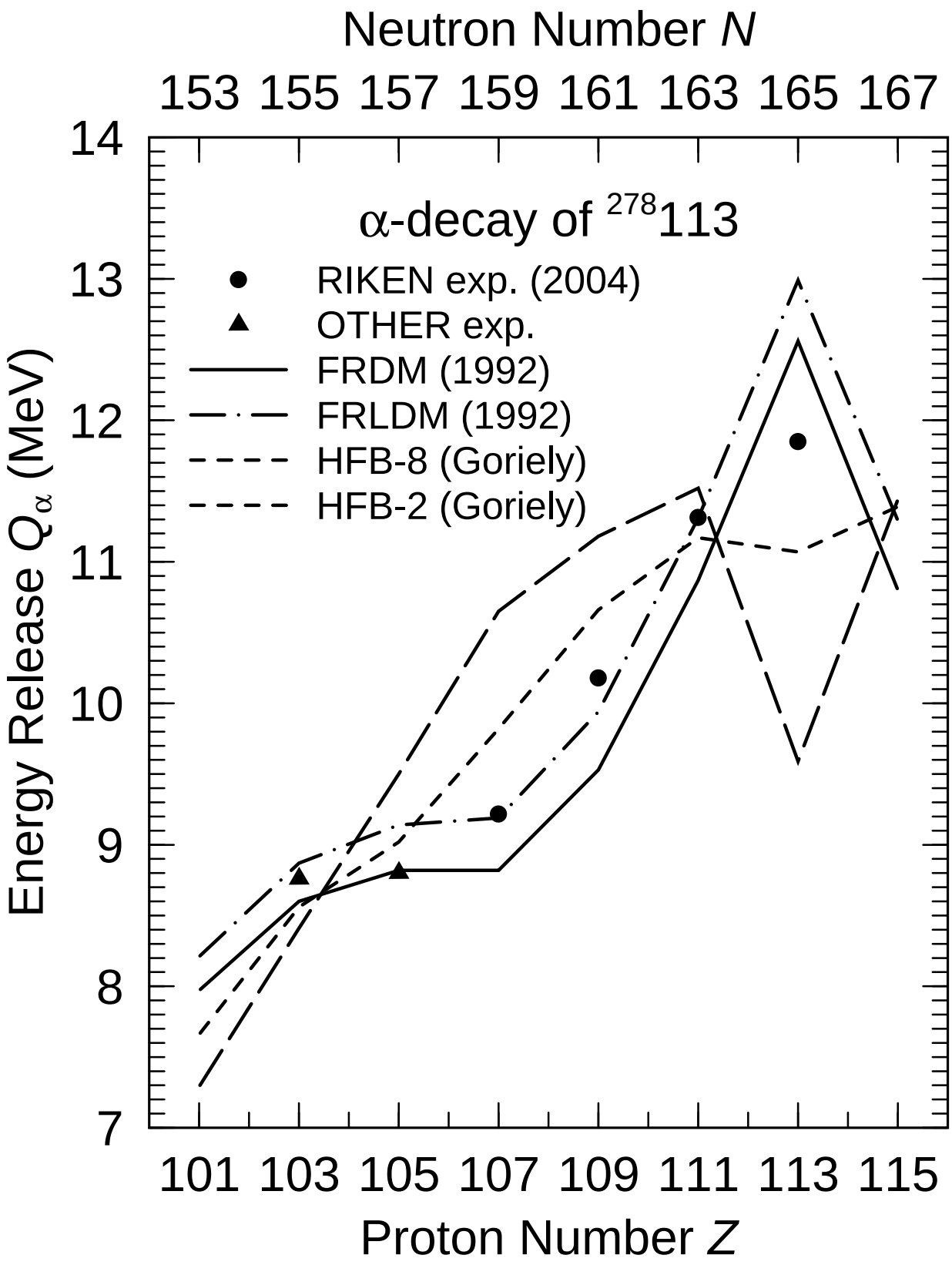

Number of Minima
β-stable nuclei
Limit AME2003
Yield limit
5
4
3
2
1
Proton Number Z
Neutron Number N
90
80
70
60
50
40
30
20
30
40
50
60
70
80
90
100
110


$^{92}_{32}Ge_{60}$
Scale 0.20 (MeV)
60
40
20
0
Axial Asymmetry γ
0.00
0.10
0.20
0.30
0.40
Spheroidal Deformation ε_2
3
4
5
6
7


$^{94}_{34}Se_{60}$
Scale 0.20 (MeV)
60
40
20
0
Axial Asymmetry γ
7
6
5
4
3
0.00
0.10
0.20
0.30
0.40
Spheroidal Deformation ε_2

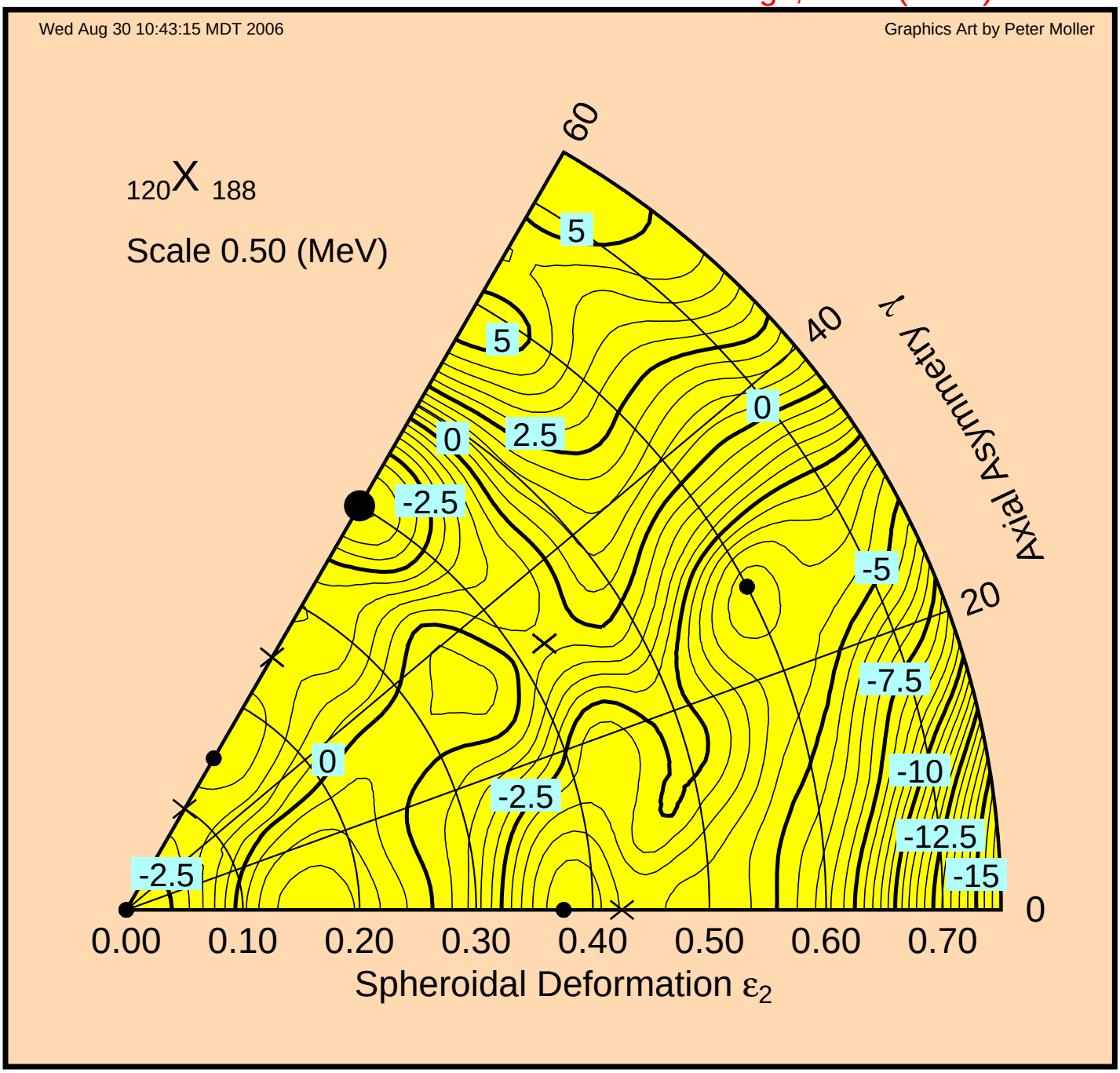

$^{93}_{33}As_{60}$
Scale 0.20 (MeV)
60
40
20
0
Axial Asymmetry γ
5
7
6
4
5
4
3
3
6
0.00
0.10
0.20
0.30
0.40
Spheroidal Deformation ε_2

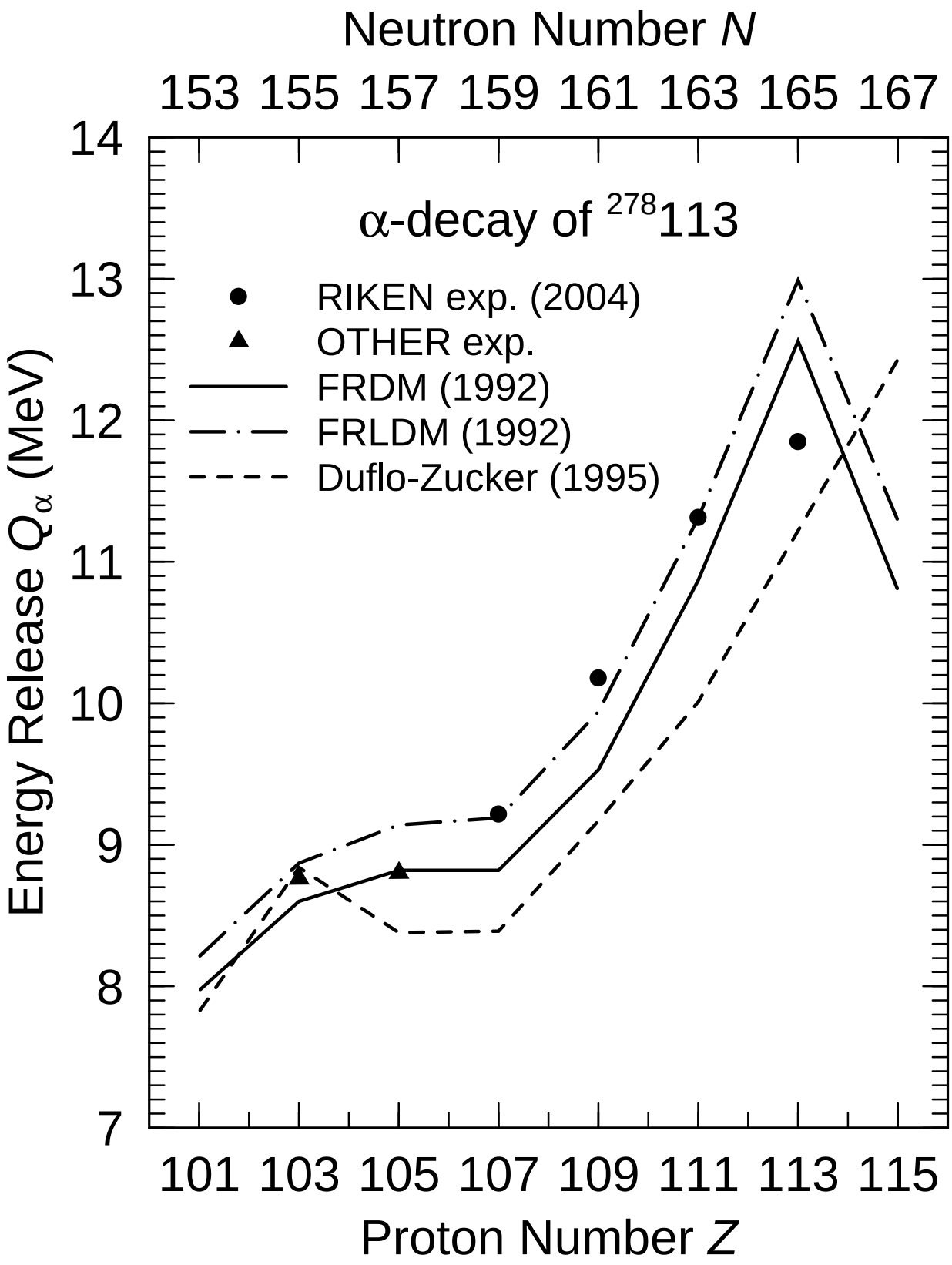

FRLDM-2007b Compared to Audi 2003
Discrepancy (Exp. – Calc.)
|ΔE| (MeV)
1.50
1.00
0.50
0.00
– 0.50
– 1.00
– 1.50
– 2.00
Proton Number Z
Neutron Number N
0
10
20
30
40
50
60
70
80
90
100
110
120
0
20
40
60
80
100
120
140
160

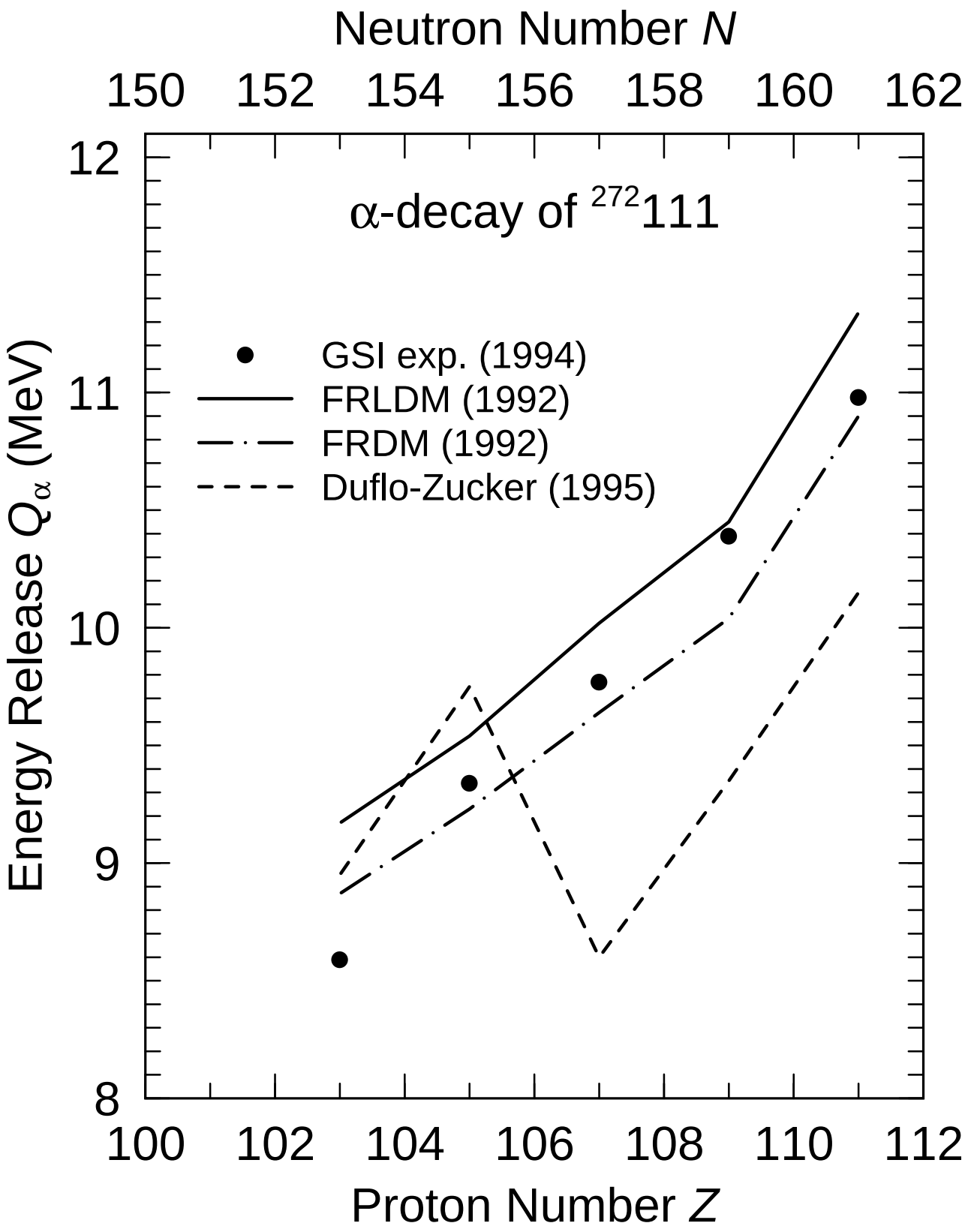


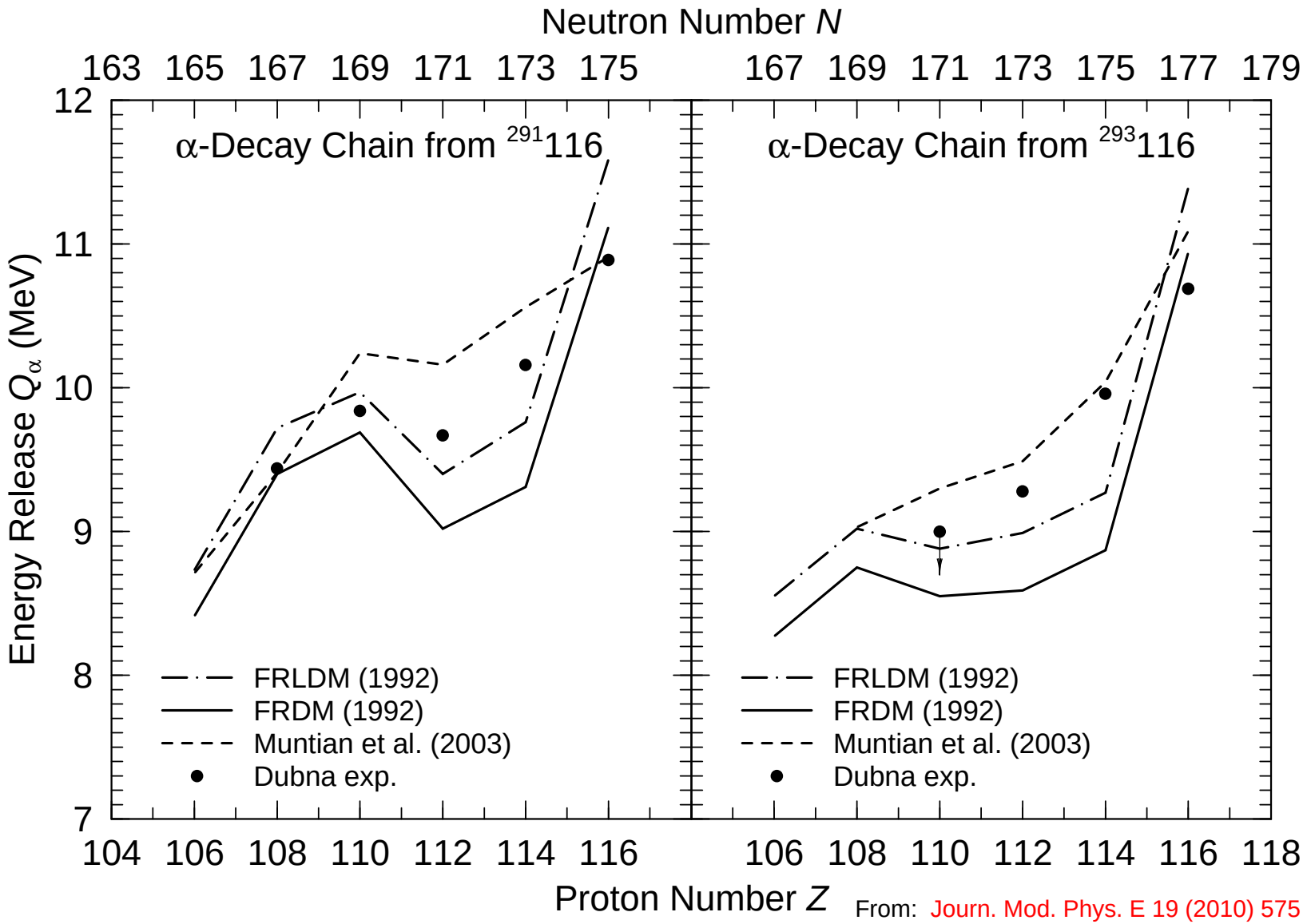
From: AIP Conference Proceedings, 1016 (2008) 15

From: AIP Conference Proceedings, 1016 (2008) 15




Neutron Number N
153 155 157 159 161 163 165 167
α-decay of 278113
RIKEN exp. (2004)
OTHER exp.
FRDM (1992)
FRLDM (1992)
Duflo-Zucker (1995)
Energy Release Qα (MeV)
14
13
12
11
10
9
8
7
101 103 105 107 109 111 113 115
Proton Number Z

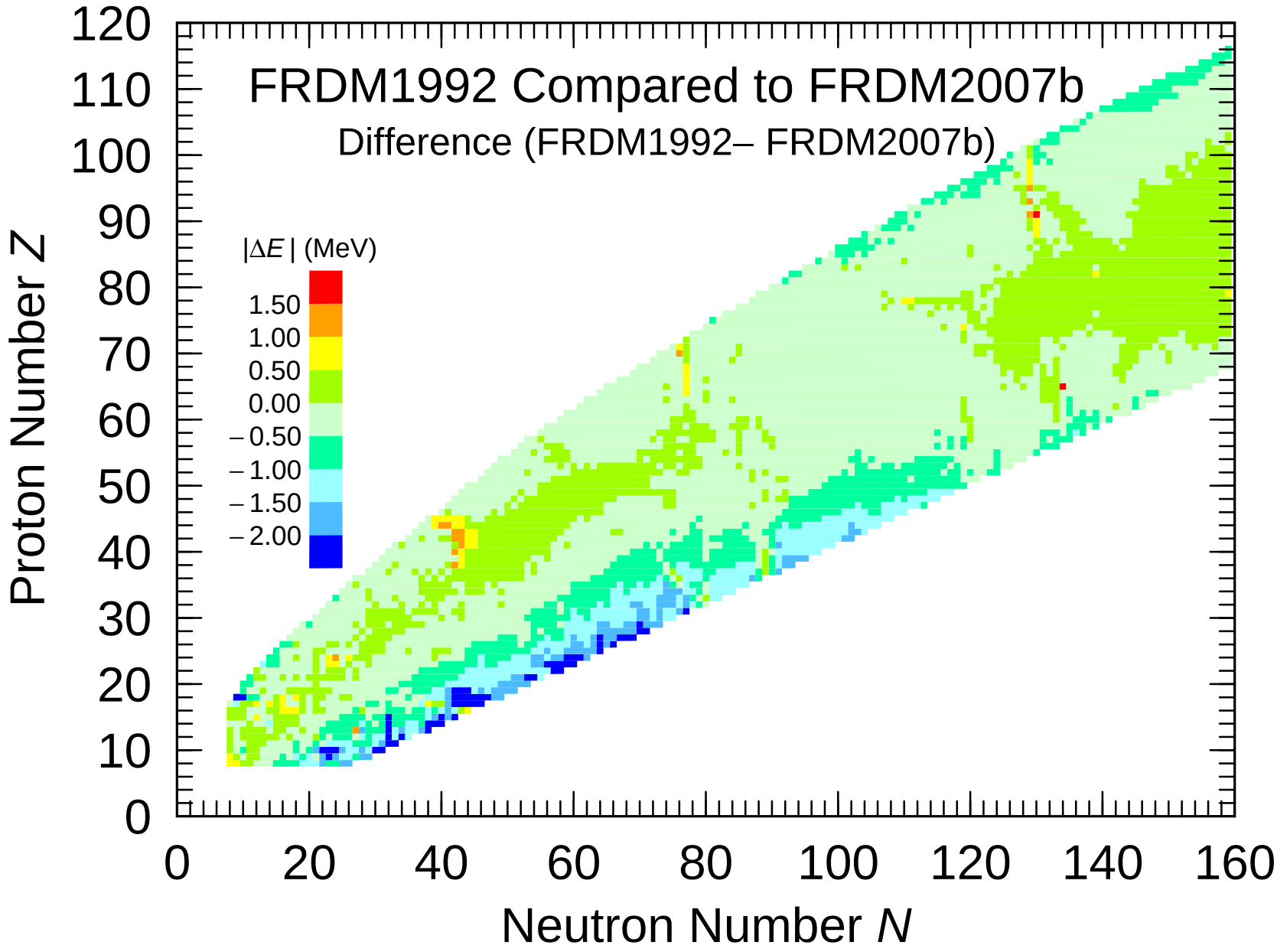

Neutron Number N
150
152
154
156
158
160
162
α-decay of 272111
GSI exp. (1994)
FRLDM (1992)
FRDM (1992)
Duflo-Zucker (1995)
Energy Release Qα (MeV)
12
11
10
9
8
100
102
104
106
108
110
112
Proton Number Z



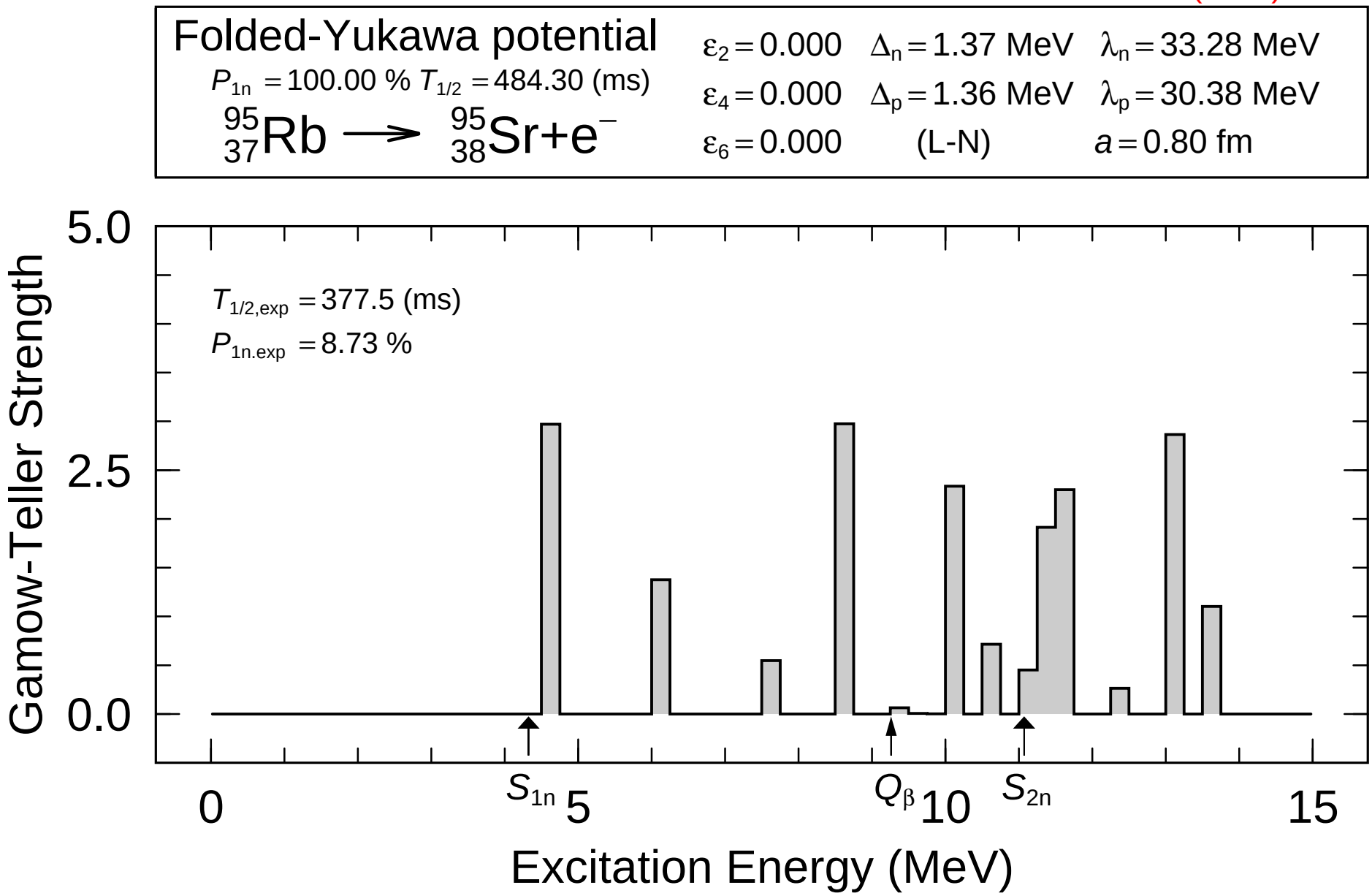
From: Journ. Mod. Phys. E 19 (2010) 575

Table 1: Comparison of trap mass measurements to FRDM1992 and FRDM07b.

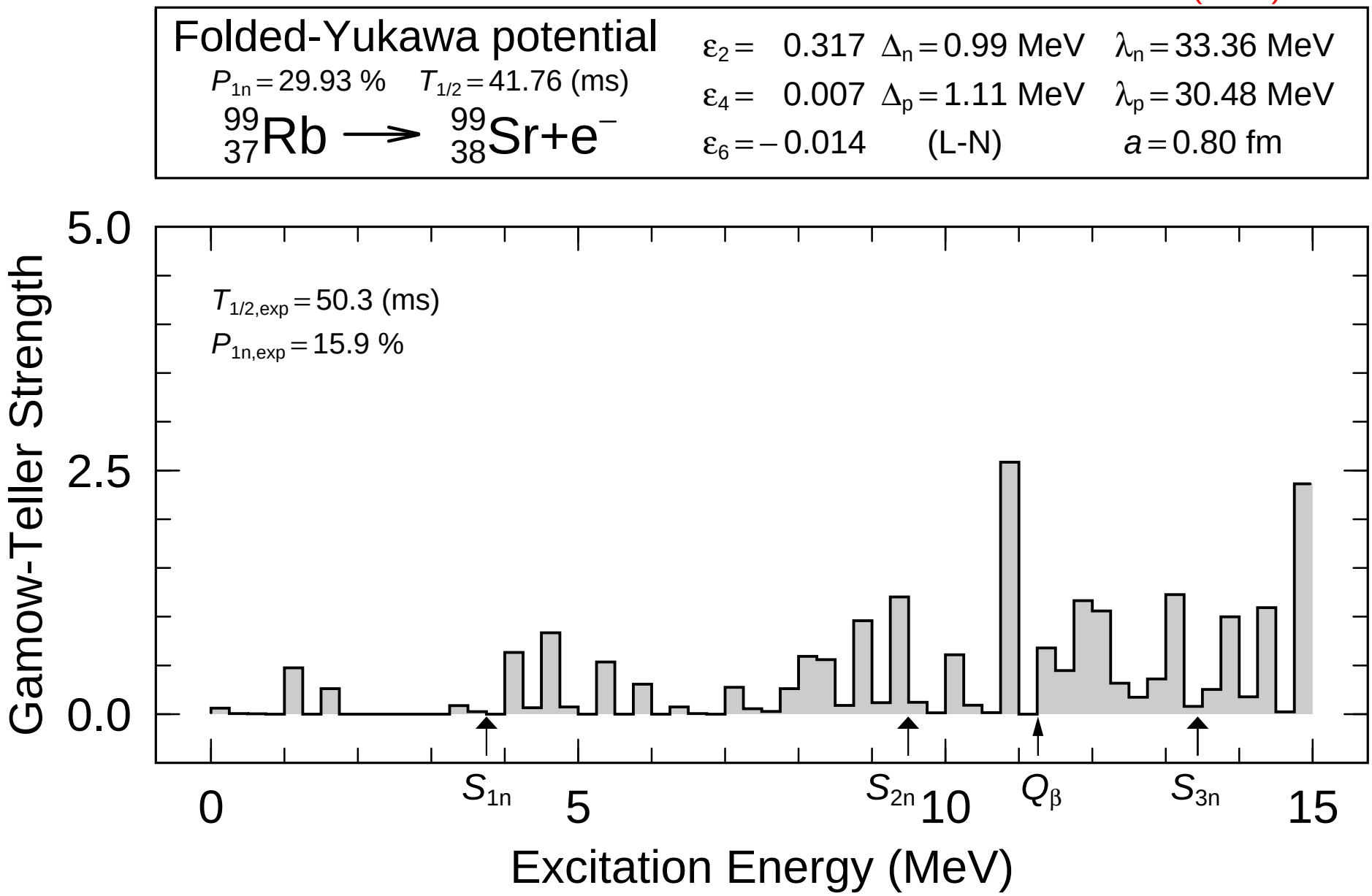
Z	A	AME2003 (MeV)	Trap (MeV)	FRDM1992 (MeV)	Dev1992 (MeV)	FRDM07b (MeV)	Dev07b (MeV)
38	80	-70.308	-70.313	-68.840	-1.473	-70.102	-0.211
38	81	-71.528	-71.528	-70.650	-0.878	-71.262	-0.266
38	84	-80.644	-80.648	-80.880	0.232	-80.984	0.336
40	86	-77.800	-77.971	-77.960	-0.011	-78.132	0.161
41	85	-67.150	-66.279	-65.350	-0.929	-66.140	-0.139
42	85	-59.100#	-57.510	-55.750	-1.760	-57.030	-0.480
42	86	-64.560	-64.110	-62.720	-1.390	-63.496	-0.614
42	87	-67.690	-66.882	-66.030	-0.852	-66.593	-0.289
43	87	-59.120#	-57.690	-56.540	-1.150	-57.403	-0.287


FRDM1992 Compared to FRDM2007b
Difference (FRDM1992– FRDM2007b)
|ΔE| (MeV)
1.50
1.00
0.50
0.00
– 0.50
– 1.00
– 1.50
– 2.00
Proton Number Z
Neutron Number N
0
10
20
30
40
50
60
70
80
90
100
110
120
0
20
40
60
80
100
120
140
160

From: At. Data Nucl. Data Tables 66 (1997) 131



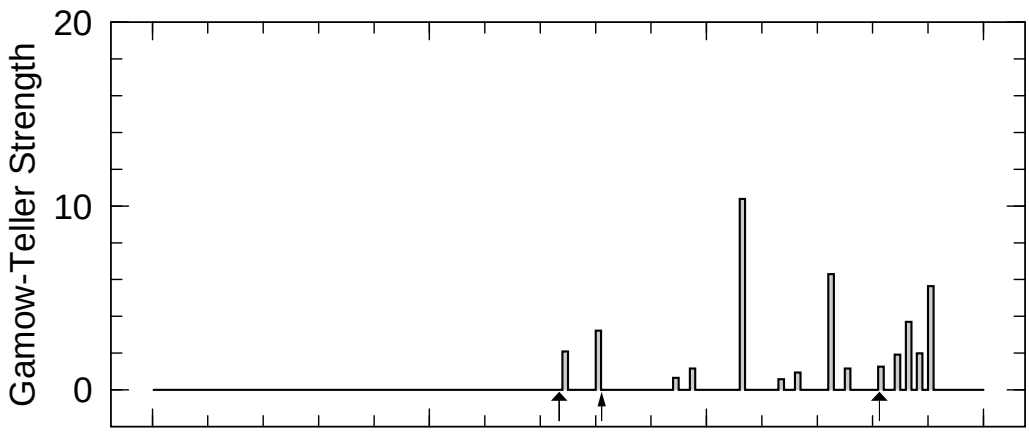
From: At. Data Nucl. Data Tables 66 (1997) 131



From: Phys. Rev. C 67 (2003) 055802

β-Decay of ^{92}Rb in 3 Successively Improved Models

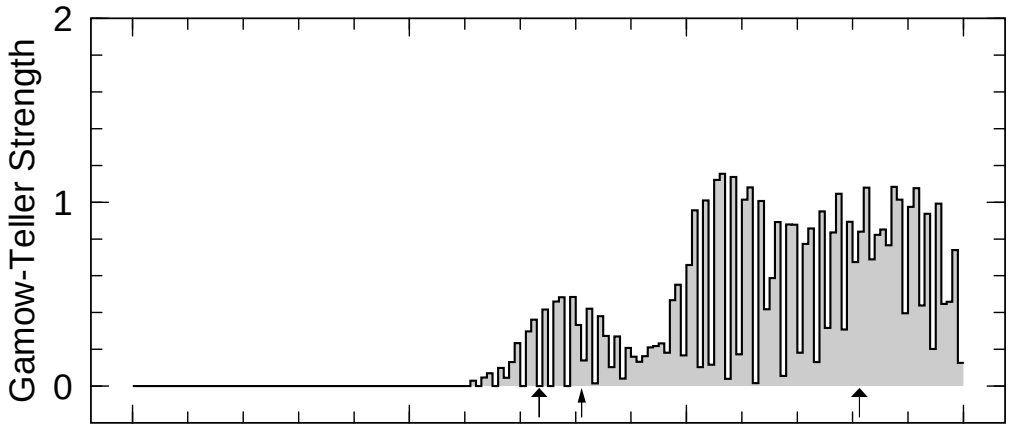
(Exp.: $T_{1/2} = 4.49$ s $P_n = 0.01$ %)



QRPA

Pure GT strength
Pure q.p. levels

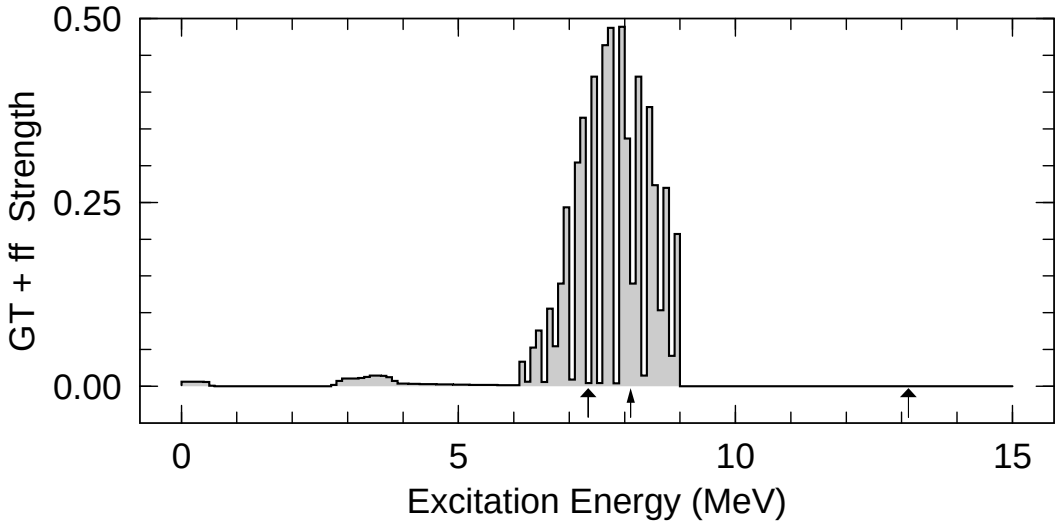
P_n = 82.81 %
$T_{1/2}$= 1.30 (h)



QRPA

Pure GT strength
Spreading of
q.p. levels

P_n = 4.14 %
$T_{1/2}$= 12.05 (min)



QRPA (GT) + Gross Th. (ff)

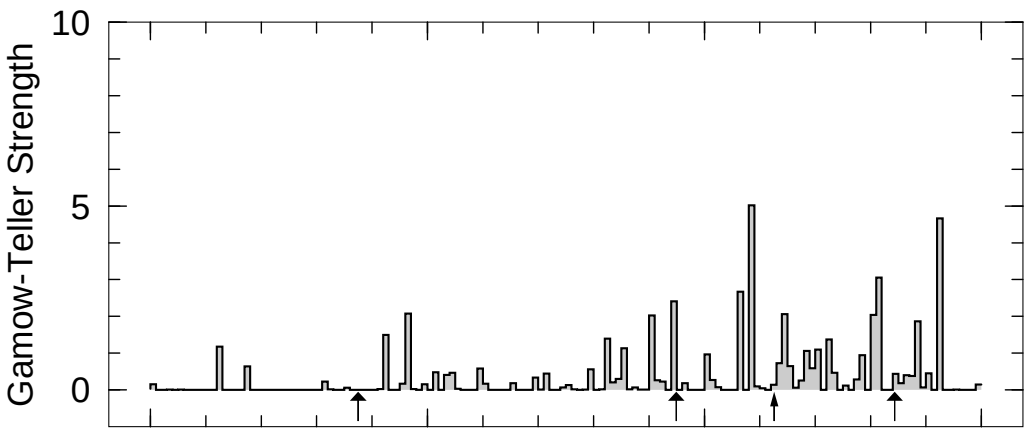
with spreading

P_n = 0.05 %
$T_{1/2}$= 8.47 (s)

From: Phys. Rev. C 67 (2003) 055802

β-Decay of ^{99}Rb in 3 Successively Improved Models

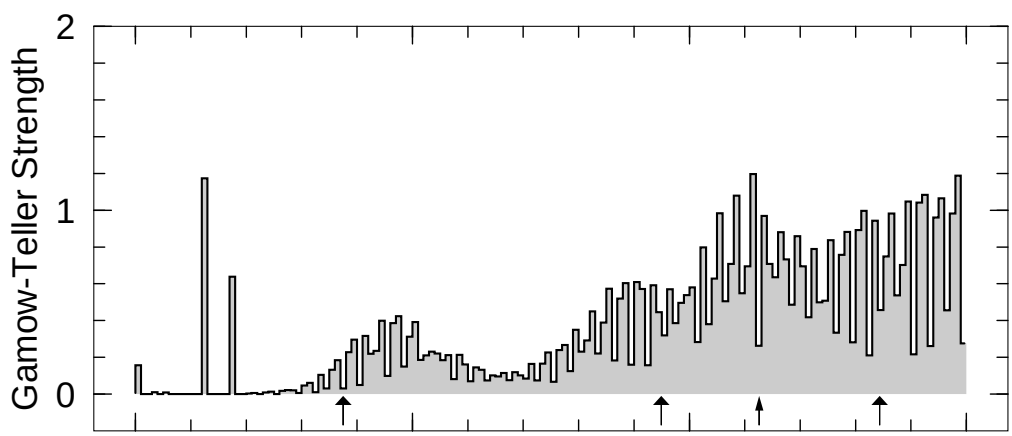
(Exp.: $T_{1/2} = 50.3$ ms $P_n = 17.3$ %)



QRPA

Pure GT strength
Pure q.p. levels

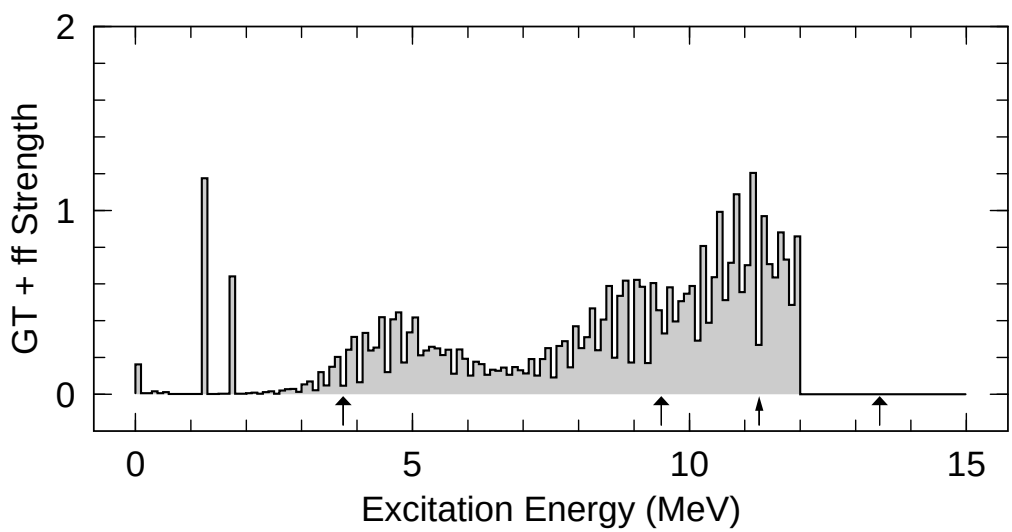
P_n = 29.53 %
$T_{1/2}$= 44.59 (ms)



QRPA

Pure GT strength
Spreading of
q.p. levels

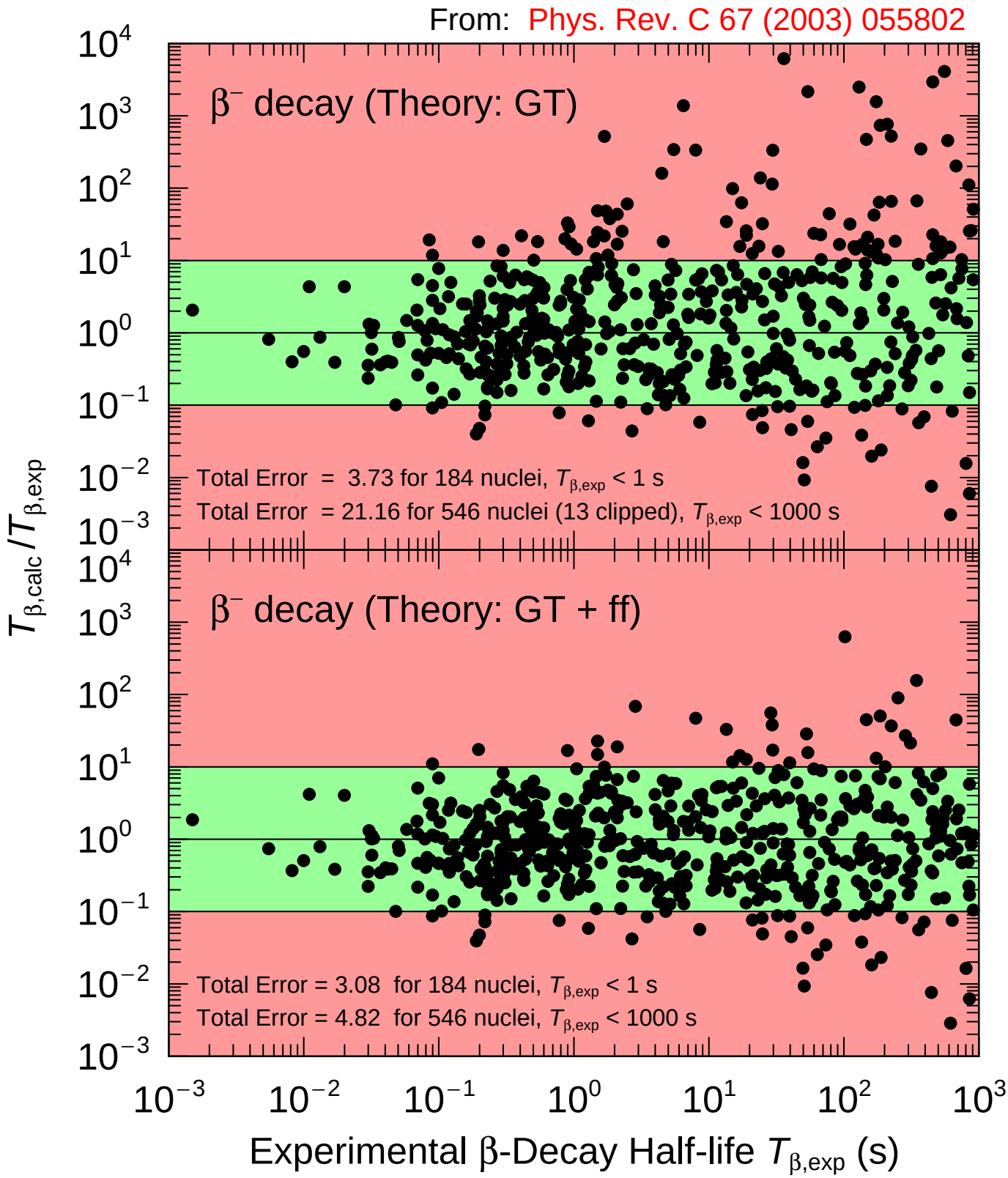
P_n = 26.88 %
$T_{1/2}$= 43.61 (ms)

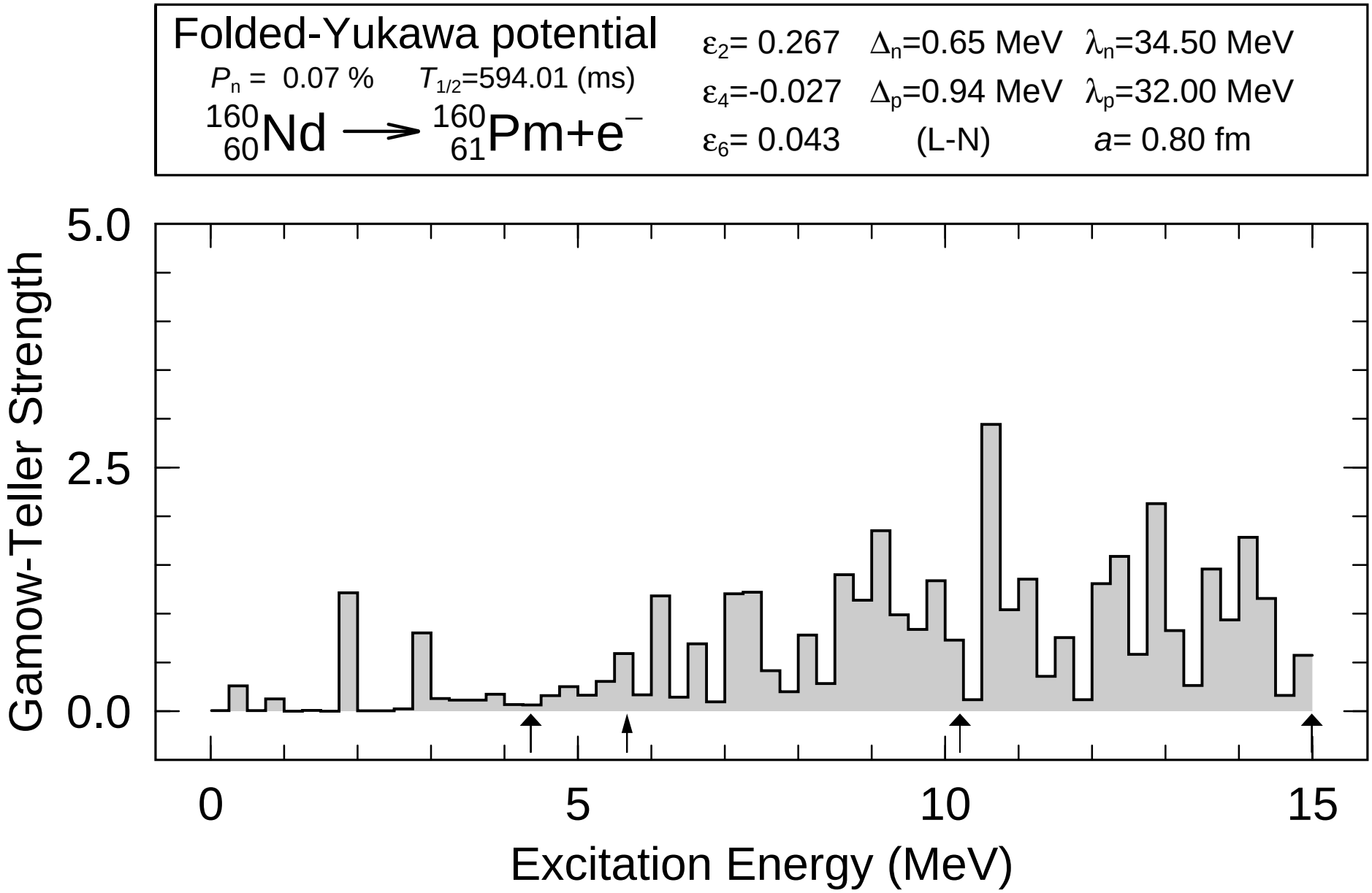


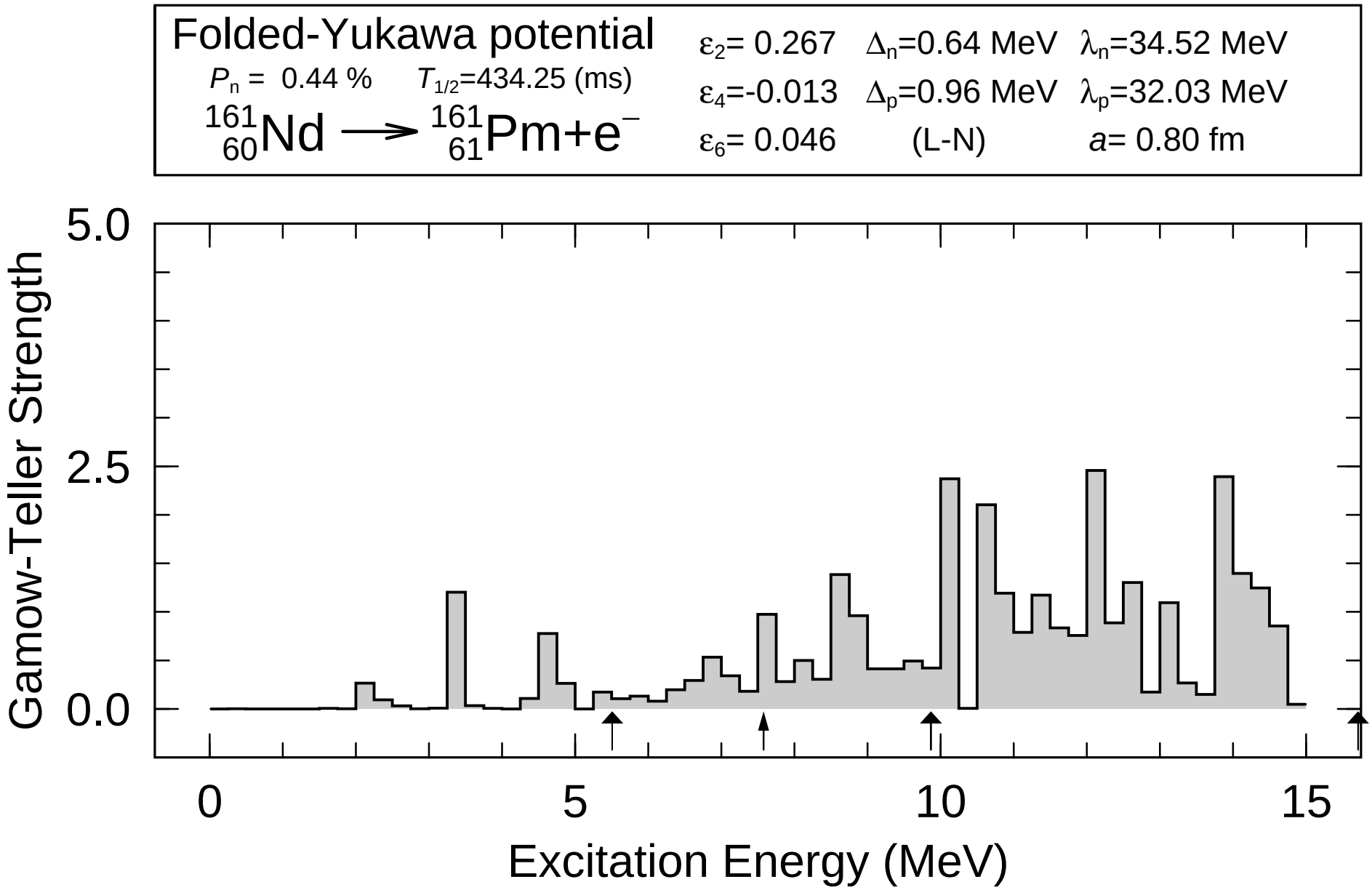
QRPA (GT) + Gross Th. (ff)

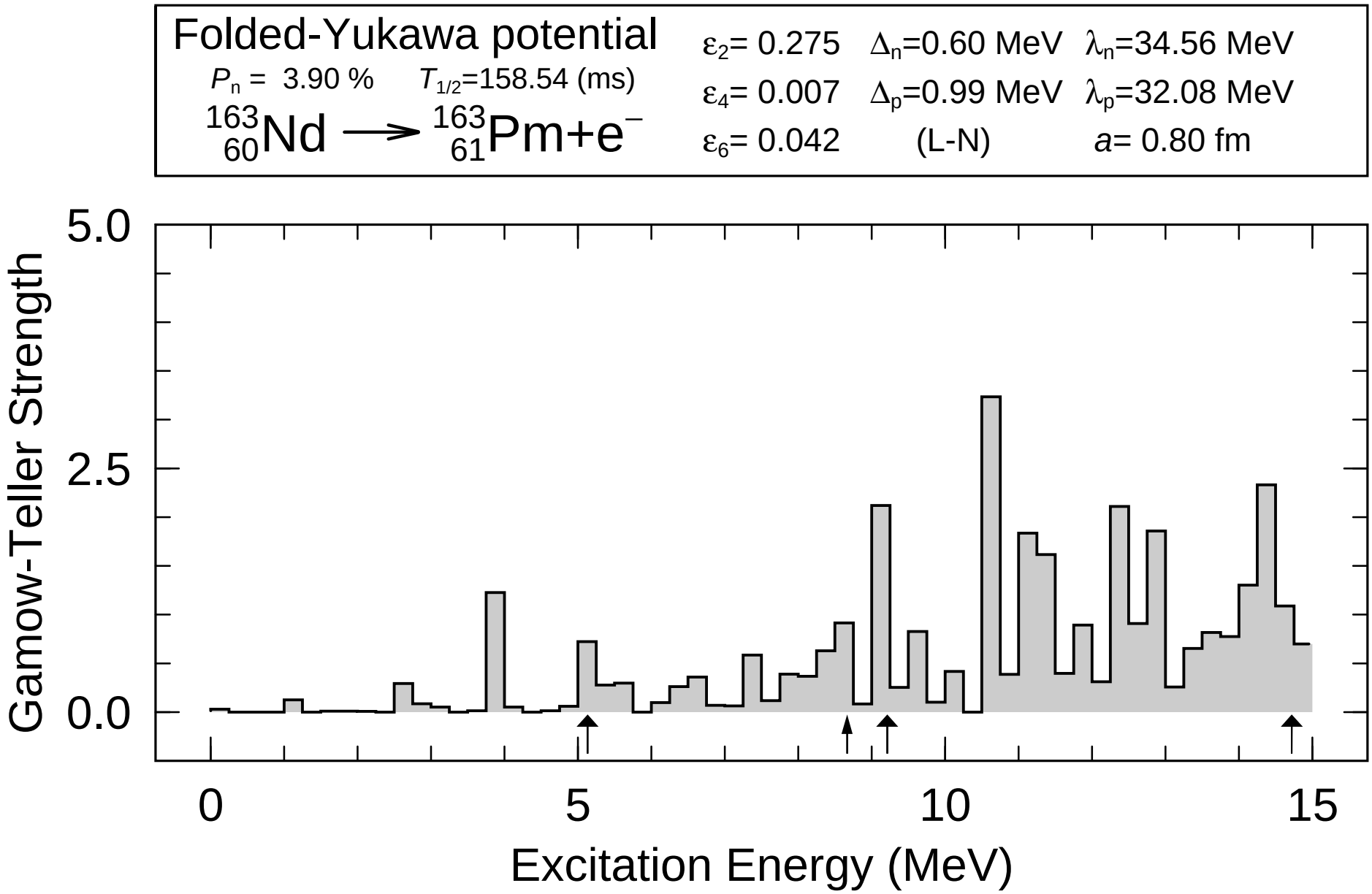
with spreading

P_n = 27.52 %
$T_{1/2}$= 40.41 (ms)


From: Phys. Rev. C 67 (2003) 055802
β⁻ decay (Theory: GT)
Total Error = 3.73 for 184 nuclei, $T_{\beta,exp}$ < 1 s
Total Error = 21.16 for 546 nuclei (13 clipped), $T_{\beta,exp}$ < 1000 s
β⁻ decay (Theory: GT + ff)
Total Error = 3.08 for 184 nuclei, $T_{\beta,exp}$ < 1 s
Total Error = 4.82 for 546 nuclei, $T_{\beta,exp}$ < 1000 s
$T_{\beta,calc}/T_{\beta,exp}$
Experimental β-Decay Half-life $T_{\beta,exp}$ (s)


Folded-Yukawa potential
P_n = 0.07 %
$T_{1/2}$=594.01 (ms)
$^{160}_{60}Nd \rightarrow ^{160}_{61}Pm + e^-$
ε_2= 0.267
ε_4=-0.027
ε_6= 0.043
Δ_n=0.65 MeV
Δ_p=0.94 MeV
(L-N)
λ_n=34.50 MeV
λ_p=32.00 MeV
a= 0.80 fm
Gamow-Teller Strength
5.0
2.5
0.0
0
5
10
15
Excitation Energy (MeV)


Folded-Yukawa potential
P_n = 0.44 %
$T_{1/2}$=434.25 (ms)
$^{161}_{60}Nd \rightarrow ^{161}_{61}Pm + e^-$
ε_2= 0.267
ε_4=-0.013
ε_6= 0.046
Δ_n=0.64 MeV
Δ_p=0.96 MeV
(L-N)
λ_n=34.52 MeV
λ_p=32.03 MeV
a= 0.80 fm
Gamow-Teller Strength
5.0
2.5
0.0
0
5
10
15
Excitation Energy (MeV)


Folded-Yukawa potential
P_n = 3.90 %
$T_{1/2}$=158.54 (ms)
$^{163}_{60}Nd \rightarrow ^{163}_{61}Pm + e^-$
ε_2= 0.275
ε_4= 0.007
ε_6= 0.042
Δ_n=0.60 MeV
Δ_p=0.99 MeV
(L-N)
λ_n=34.56 MeV
λ_p=32.08 MeV
a= 0.80 fm
Gamow-Teller Strength
5.0
2.5
0.0
0
5
10
15
Excitation Energy (MeV)

Z = 60 A = 163
Pn = 3.901 EQBET = 8.66MeV B1n = 5.13 MeV
T-1/2 (SECS) = 0.158537E+00 EQBET = 8.6650 MeV

Ener	Beta Strength	IP	IN	SIG	OMP	OMN	DV	Int. (%)	log(ft)
0.00	0.7126492E-02	31	52	1	5/2-	5/2-	0	8.7092	5.7632
1.07	0.3129956E-01	34	52	2	7/2-	5/2-	0	21.3281	5.1205
1.63	0.2356034E-02	31	52	1	5/2-	5/2-	2	1.1467	6.2439
1.88	0.2228313E-02	30	50	2	3/2-	1/2-	2	0.9256	6.2681
2.09	0.5803861E-03	29	50	2	1/2-	1/2-	2	0.2101	6.8523
2.10	0.1159219E-02	29	50	1	1/2-	1/2-	2	0.4191	6.5519
2.59	0.6225789E-01	31	49	1	5/2-	5/2-	2	15.9430	4.8219
2.65	0.2050320E-02	30	49	2	3/2-	5/2-	2	0.5053	6.3042
2.71	0.9116088E-02	34	52	2	7/2-	5/2-	2	2.1463	5.6562
2.92	0.2137760E-01	31	47	2	5/2-	3/2-	2	4.2976	5.2861
3.02	0.7953920E-02	30	47	1	3/2-	3/2-	2	1.4845	5.7155
3.22	0.6947581E-03	29	47	2	1/2-	3/2-	2	0.1104	6.7742
3.23	0.4123438E-02	34	53	1	7/2-	7/2-	2	0.6499	6.0008
3.69	0.3625175E-02	40	52	2	3/2-	5/2-	0	0.3927	6.0567
3.75	0.3067371E+00	31	58	2	5/2-	3/2-	2	31.3783	4.1293
4.01	0.7692047E-02	36	50	1	1/2-	1/2-	2	0.6212	5.7300
4.01	0.5194031E-02	34	49	2	7/2-	5/2-	2	0.4215	5.9006
4.55	0.3513152E-02	36	50	2	1/2-	1/2-	2	0.1676	6.0704
4.90	0.1451347E-01	26	60	1	3/2+	3/2+	2	0.4775	5.4543
5.00	0.9234226E-01	32	43	2	5/2+	3/2+	2	2.6928	4.6507
5.07	0.2598458E-01	31	41	2	5/2-	3/2-	2	0.7030	5.2013
5.07	0.2591038E-01	33	43	1	3/2+	3/2+	2	0.6962	5.2026
5.13	0.6271024E-02	33	42	2	3/2+	1/2+	2	0.1576	5.8187
5.15	0.3014728E-01	30	40	2	3/2-	1/2-	2	0.7436	5.1368
5.26	0.4791617E-03	36	47	2	1/2-	3/2-	2	0.0103	6.9356
5.28	0.3418437E-01	29	41	2	1/2-	3/2-	2	0.7175	5.0822
5.32	0.3216554E-02	39	53	2	9/2-	7/2-	2	0.0644	6.1087
5.34	0.1065636E-01	40	52	2	3/2-	5/2-	2	0.2072	5.5884
5.34	0.6631651E-02	27	42	1	1/2+	1/2+	2	0.1283	5.7944
5.47	0.1427046E-01	29	40	2	1/2-	1/2-	2	0.2340	5.4616
5.57	0.3248029E-01	40	50	2	3/2-	1/2-	2	0.4658	5.1044
5.60	0.4160867E-01	30	41	1	3/2-	3/2-	2	0.5742	4.9969

SUMRULE 99.9999924 99.9999924

ONGOING NEAR-TERM PROJECTS

- A global study of nuclear shape isomers will appear shortly in ADNDT (is available on my web site).
- We have calculated 5D potential-energy surfaces for 5254 nuclei for 5009325 different shapes (PRC **79** (2009) 064304)
- Recently we have developed a model of fission yields (PRL **106** (2011) 132503) that can straightforwardly be applied to the 5254 nuclei.
- We are finalizing a combinatorial level-density model with folded-Yukawa single-particle levels as a starting point. The model will allow us to calculate level densities at the exact barrier shapes we have calculated. Preliminary results are in PRC **79** (2009) 064304.
- Improve the yield model to take into account the metric of the deformation grid.
- Improve mass model accuracy for light nuclei. We probably need new term in the single-particle potential over and above the spin-orbit.
- Apply to astrophysical network calculations (r-process, neutron star, etc)

SUMMARY

- We have presented some results from a general nuclear-structure model that can globally provide a large number of nuclear-structure quantiles, more than have currently been explored.
- In particular, nuclear mass calculations agrees on the average with new mass data to better accuracy than in the region where model parameters were determined.
- We have added successive improvements to the mass model, the current (unpublished) version 2009b had an accuracy of 0.5788 MeV with respect to AME2003.
- It remains to be seen if the model accuracy can be improved for light nuclei ($N \leq 60$). Possibly a new term in the single-particle interaction (over and above the spin-orbit interaction) is needed.
- The Caribu effort will provide several different types of experimental data in particular above $N = 60$ where the model is expected to do well. It will be highly valuable to test the model and this assumption on this forthcoming experimental data far from stability, with an eye on understanding what accuracy we can expect from our models in the r-process region.

RELATED PAPERS, FIGURES, AND TABLES ARE AT:

http://t16web.lanl.gov/Moller/abstracts.html