Sample-Size Effects in Fast-Neutron Gamma-Ray Production Measurements: Solid-Cylinder Samples

by

Donald L. Smith

September 1975

ARGONNE NATIONAL LABORATORY,
ARGONNE, ILLINOIS 60439, U.S.A.
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In January 1975, the research and development functions of the former U.S. Atomic Energy Commission were incorporated into those of the U.S. Energy Research and Development Administration.

Applied Physics Division
Argonne National Laboratory
9700 South Cass Avenue
Argonne, Illinois 60439
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SAMPLE-SIZE EFFECTS IN
FAST-NEUTRON GAMMA-RAY
PRODUCTION MEASUREMENTS:
SOLID-CYLINDER SAMPLES*

by

Donald L. Smith

Argonne National Laboratory, Argonne, Illinois 60439, U.S.A.

ABSTRACT

The effects of geometry, absorption and multiple scattering in \( (n,\text{Xy}) \) reaction measurements with solid-cylinder samples are investigated. Both analytical and Monte-Carlo methods are employed in the analysis. Geometric effects are shown to be relatively insignificant except in definition of the scattering angles. However, absorption and multiple-scattering effects are quite important; accurate microscopic differential cross sections can be extracted from experimental data only after a careful determination of corrections for these processes. The results of measurements performed using several natural iron samples (covering a wide range of sizes) confirm validity of the correction procedures described herein. It is concluded that these procedures are reliable whenever sufficiently accurate neutron and photon cross section and angular distribution information is available for the analysis.

*This work was performed under the auspices of the U.S. Energy Research and Development Administration.
1. INTRODUCTION

A recent report describes the facility which has been developed at Argonne National Laboratory's FNG for (n,XY) reaction studies [1,2]. The geometry is shown in Figs.1-6 of Ref. 1 and it is recommended that the reader refer to this earlier report in conjunction with the present one.

Experience with this facility has indicated that the precision of raw data obtained generally improves with increased sample size. The relative importance of background decreases and the statistical accuracy of the significant data improves under these conditions. Furthermore, in measurements with relatively large samples, it is possible to exploit the advantages of longer flight paths (improved time-of-flight resolution) and massive detector shielding. The penalty involved in measurements with large samples is that corrections to the raw data for effects of absorption and multiple scattering can be quite large. Accurate determination of these corrections requires knowledge of neutron and photon cross sections and their angular distributions and use of complex computational procedures. Clearly, a compromise is necessary. The objective of this report is to present the results of a detailed study of sample-size effects which was conducted in the course of developing the data processing routines which are employed in the reduction of experimental data acquired with this facility.

The results of a literature survey were disappointing. There are relatively few readily available articles on the subject of sample corrections [3-13]. Most of these references deal with experiments in which neutrons (not gamma rays) are detected. These articles provided guidance, but were not a basis for the present work.

The analysis presented in this report employs only a few simplifying assumptions and therefore adheres to a
realistic representation of the physical problem.

The $^7\text{Li}(p,n)^7\text{Be}$ reaction is usually used as a neutron source at this laboratory for measurements in the region of interest for current $(n,X\gamma)$ studies ($E_n \lesssim 5$ MeV). Natural lithium metal is evaporated onto a thin tantalum backing to form a target. The proton-beam spot on target is defined by slits and is essentially rectangular. The lithium films are relatively thin ($\Delta E_p \lesssim 0.1$ MeV). The analysis presented in this report assumes an infinitesimal-thin square target; however, the routines actually used for data processing take cognizance of realistic target thicknesses by superimposing contributions from several very thin layers. Target thickness is an important consideration whenever the cross section varies rapidly with neutron energy or for proton energies near the resonance in the lithium source reaction at $\sim 2.3$ MeV. The angular distribution of neutrons from the source reaction is taken into account. Neutrons from the $^7\text{Li}(p,n)^7\text{Be}^*$ and $^7\text{Li}(p,n)^3\text{He}^4\text{He}$ reactions are considered in the analysis for proton energies above their production thresholds.

Gamma rays from $(n,X\gamma)$ reactions are detected with a $\text{Ge(Li)}$ detector and the yields of full-energy-peak events are recorded. Since the $\text{Ge(Li)}$ detector has a diameter of $\sim 5$ cm and is placed $\sim 100$ to $150$ cm from the sample, it is assumed that the only photons which are capable of producing full-energy pulses in the detector are those which either experience no interaction in the sample after production via $(n,X\gamma)$ reactions or are coherently scattered in the sample. Therefore, the total photon cross section is assumed for sample absorption calculations and a small correction for coherent scattering is applied when necessary (see Appendix A).

The scattering of neutrons in the sample by elastic and inelastic scattering through discrete levels is treated.
The effects of more complicated reactions such as \((n;2n)\), \((n;n,p)\), \((n;n,p)\), \((n;n,a)\), etc. are insignificant for \(E \lesssim 5\) MeV so they are ignored in the computations. Kinematic effects are considered, and energy-dependent cross sections and angular distributions are employed.

The scattering sample is assumed to be a uniform right-circular cylinder centered on the beam line with axis normal to the scattering plane defined by the beam line and the detector. Macroscopic cross sections are used in absorption calculations, and samples consisting of either single- or multiple-isotope elements, compounds or mixtures can be treated.

Let

\[
Y_{\text{TOT}} = \text{total observed gamma-ray yield for a particular geometry,}
\]

\[
Y_0 = \text{gamma-ray yield produced by unscattered neutrons,}
\]

\[
Y_k = \text{gamma-ray yield produced by neutrons which have scattered "}k\text{" times in the sample before initiating (n,\gamma) reactions,}
\]

\[
k = \text{highest order of scattering considered,}
\]

then

\[
Y_{\text{TOT}} \lesssim Y_0 + \sum_{k=1}^{k} Y_k = Y_0 [1 + \sum_{k=1}^{k} (Y_k/Y_0)]. \tag{1}
\]

If

\[
\alpha_k = (Y_k/Y_0),
\]

\[
\alpha_{\text{TOT}} = \sum_{k=1}^{k} \alpha_k, \tag{3}
\]

then

\[
Y_{\text{TOT}} \lesssim Y_0 (1 + \alpha_{\text{TOT}}). \tag{4}
\]

The symbol \(\sum\) represents summation to avoid confusion with \(\Sigma\) used for macroscopic cross sections elsewhere in this report. The quantity \(\alpha_{\text{TOT}}\) is called the multiple-
scattering parameter and $\alpha_z$ is the $z$-th component. For most samples,

$$\alpha_{\text{TOT}} < 1 \text{ and } \alpha_{k+1} \ll \alpha_z,$$

and the sums in Eqs. 1 and 3 converge rapidly. Acceptable accuracy is obtained in practice for $k \leq 3$.

The evaluation of $Y_0$ and $\alpha_{\text{TOT}}$ are treated separately. The quantity $Y_0$ is determined analytically while $\alpha_{\text{TOT}}$ is deduced by Monte-Carlo methods.

The acquisition of nuclear data required for the computations is considered in Section 2; this is a problem which must be addressed before any sample-size correction factors can be computed. Section 3 of this report deals with evaluation of $Y_0$ and its relationship to the $(n, X_y)$ reaction differential cross section which is sought from the measurements. Section 4 treats the subject of multiple scattering and determination of $\alpha_{\text{TOT}}$. In Section 5, the results of computations are compared with experimental data and a simplified computational procedure is explored.

2. NUCLEAR DATA FOR COMPUTATIONS

The methods for computation of sample correction factors described in this report are powerful in principle; however, the quality of the results obtained is only as good as the accuracy of the nuclear data utilized in the analysis. Thus, the experimenter must exercise judgement in selection of the sample size for an experiment. If the available cross section and angular distribution information is uncertain, it is necessary to use smaller samples and thereby sacrifice sensitivity and statistical accuracy in order to minimize the magnitude of the corrections which must be computed. Actually, the quality of available nuclear data is gradually improving, and high-
speed digital computers are accessible to most researchers. Therefore, it appears worthwhile to develop the sophisticated computational tools required to determine realistic corrections and exploit the experimental advantages of using relatively large samples whenever possible.

The existence of pronounced resonance structure in nuclear data complicates many aspects of nuclear science and technology; the present topic is no exception. It has been found convenient to smooth all energy-dependent nuclear data used in correction calculations with resolution functions which approximate the experimental conditions. The smoothed excitation functions can usually be represented with sufficient accuracy by a relatively small number of parameters. Fig. 1 demonstrates the concept. The use of smoothed cross sections for sample absorption and multiple-scattering calculations is an approximation, the validity of which must be investigated carefully prior to use in applications. One method is to compare the results of small- and large-sample measurements in regions where strong resonances are present in the cross sections.

Neutron cross section and angular distribution information is obtained from the evaluated neutron data file, ENDF/B-IV [14]; photon cross sections are obtained from an evaluation by Storm and Israel [15].

3. EFFECTS OF GEOMETRY AND ABSORPTION

The dominant features of the observed gamma-ray yield from \((n,X\gamma)\) reactions in the sample are determined by geometry and the absorption of neutron and gamma radiation. These features are predicted by the response of \(Y_0\) in Eq. (4) to variation of the experimental conditions. Multiple scattering, represented by \(\alpha_{\text{TOT}}\) in Eq. (4), yields a less significant correction to this behavior. This section describes the procedure used to compute \(Y_0\) and presents the results of calculations designed to explore the sensitivity
of \( Y_0 \) to various parameters.

3.1 Mathematical Formalism

The yield for the entire sample is computed by summing the contributions from various portions of the sample. The neutron source and the sample are represented as described below. Fig. 2. illustrates the geometry.

Let

\[
\begin{align*}
    d_{n} & = \text{distance from a particular neutron source point to a particular sample point,} \\
    d_{\gamma} & = \text{distance from a particular sample point to the gamma-ray detector (assumed to be a single point),} \\
    \delta_{n} & = \text{distance through the sample which the neutron must penetrate to reach the particular sample point,} \\
    \delta_{\gamma} & = \text{distance through the sample which the gamma ray must penetrate to reach the gamma-ray detector,} \\
    R_S & = \text{radius of the sample,} \\
    H & = \text{height of the sample,} \\
    D_n & = \text{distance from the center of the neutron source to the center of the sample (which is also the pivot for the gamma-ray detector),} \\
    D_{\gamma} & = \text{distance from the center of the sample to the gamma-ray detector,} \\
    \Theta_n & = \text{incident-neutron angle relative to the beam line,} \\
    \Theta_{\text{DET}} & = \text{angle of gamma-ray detector relative to the beam line,} \\
    \Theta_{n\gamma} & = \text{angle of emission of the gamma ray relative to the incident neutron,} \\
    F_n & = \text{absolute neutron-source strength of a uniform square source (neutrons/sr),} \\
    a & = \text{dimension of the square neutron source,}
\end{align*}
\]
\[(d\sigma/d\Omega)_\gamma = \text{differential gamma-ray production cross section for the (n,\gamma) reaction,}\]

\[(x_T,y_T,z_T) = \text{coordinates for a point on the neutron-source surface,}\]

\[(x_1,y_1,z_1) = \text{coordinates for a point S}_1 \text{ in the sample,}\]

\[(x_D,y_D,z_D) = \text{coordinates for the gamma-ray detector,}\]

\[N_S = \text{number of atoms per unit volume of the sample which can contribute to (n,\gamma) reactions,}\]

\[\Sigma_{nT} = \text{neutron macroscopic total cross section for the sample material,}\]

\[\Sigma_{\gamma T} = \text{photon macroscopic total cross section for the sample material.}\]

Two-body neutron-producing reactions are assumed in the present analysis. The neutron fluence \(F_n\) is a function of the reaction parameters, incident energy, and emission angle. Kinematics governs the variation of neutron energy with angle. The cross sections \((d\sigma/d\Omega)_\gamma\), \(\Sigma_{nT}\) and \(\Sigma_{\gamma T}\) are energy-dependent; \((d\sigma/d\Omega)_\gamma\) also varies with angle \(\theta_{n\gamma}\).

The grid systems for the neutron source and the sample are illustrated in Fig. 2. The sample grid system used yields more uniformly sized elements than a standard cylindrical-coordinate grid system. The parameters which define these grid systems are

\[n_H = \text{mesh for the sample height (the sample is divided into } n_H \text{ layers of height } H/n_H \text{ along the x-axis)},\]

\[n_R = \text{radial mesh (the sample is divided into a series of } n_R - 1 \text{ shells of thickness } R_S/n_R \text{ plus a central cylinder with radius } R_S/n_R),\]

\[n_\phi = \text{fundamental angular mesh (the central cylinder, } j=1, \text{ is divided into } n_\phi \text{ wedges while}\]
the jth shell has \( j n_\phi \) segments, \( j=2,\ldots,n_R \),

\( n_T = \) neutron-source mesh (the square source is
divided into \( n_T^2 \) sections each with area
\( a^2/n_T^2 \)).

The total number of elements in the sample is \( \frac{1}{2} n_H n_\phi n_R (n_R + 1) \).
The variation in volume of these elements depends only on
the radial variable. Therefore

\[
V_j = \frac{\pi R_s^2 H}{n_R^2 n_H n_\phi} \left( \frac{2j-1}{j} \right), \ j=1,\ldots,n_R.
\] (6)

A constraint on the angular mesh \( n_\phi \), required for the
type of sample grid used, is \( n_\phi \geq 2 \).

The intercept of the beam line and neutron source
plane is selected as the origin of coordinates. The
coordinates of the center of each neutron-source element
are given by \((x_{Ta}, y_{TB}, z_T)\) where

\[
x_{Ta} = \frac{a}{2n_T} (2\alpha - n_T - 1), \ \alpha = 1,\ldots,n_T,
\] (7)

\[
y_{TB} = \frac{a}{2n_T} (2\beta - n_T - 1), \ \beta = 1,\ldots,n_T,
\] (8)

and \( z_T = 0 \) for all elements.

The coordinates of the gamma-ray detector are given
by the formulas

\[
y_D = D_Y \sin \Theta_{DET},
\] (9)

\[
z_D = D_n + D_Y \cos \Theta_{DET},
\] (10)

and \( x_D = 0 \).

The coordinates of the center of each sample element
are given by the formulas

\[
x_{1i} = \frac{H}{2n_H} (2i - n_H - 1), \ i=1,\ldots,n_H,
\] (11)

\[
y_{1jk} = r_j \sin \phi_{jk}, \ j=1,\ldots,n_R \text{ and } k=1,\ldots,n_\phi.
\] (12)
\[ z_{ijk} = d_n + r_j \cos \phi_{jk}, \quad j=1, \ldots, n_R \quad \text{and} \]
\[ k=1, \ldots, jn_\phi, \] (13)

and
\[ r_j = \frac{R_S}{2n_R} (2j-1), \quad j=1, \ldots, n_R, \] (14)
\[ \phi_{jk} = \frac{\pi}{jn_\phi} (2k-1), \quad j=1, \ldots, n_R \quad \text{and} \quad k=1, \ldots, jn_\phi \] (15)

Analytic geometry is applied in derivation of formulas for \( \delta_n \) and \( \delta_\gamma \). The expression for \( \delta_n \) is
\[ \delta_n = d_n - s_n, \] (16)

where
\[ d_n = [(x_1-x_T)^2 + (y_1-y_T)^2 + (z_1-z_T)^2]^{\frac{1}{2}}, \] (17)
\[ s_n = \frac{[B_n(y_0-y_T) + C_n(z_0-z_T)]}{(B_n^2 + C_n^2)} - \frac{\{(B_n^2 + C_n^2)R_S^2 - [C_n(y_T-y_0)^2 + B_n(z_T-z_0)^2]\}^{\frac{1}{2}}}{(B_n^2 + C_n^2)}, \] (18)
\[ B_n = (y_1-y_T)/d_n, \] (19)
\[ C_n = (z_1-z_T)/d_n, \] (20)

and \((x_0, y_0, z_0)\) are the coordinates of the center of the sample \((x_0 = y_0 = 0, \quad z_0 = d_n)\). The expression for \( \delta_\gamma \) is
\[ \delta_\gamma = d_\gamma - s_\gamma, \] (21)

where
\[ d_\gamma = [(x_1-x_D)^2 + (y_1-y_D)^2 + (z_1-z_D)^2]^{\frac{1}{2}}, \] (22)
\[ s_\gamma = \frac{[B_\gamma(y_0-y_D) + C_\gamma(z_0-z_D)]}{(B_\gamma^2 + C_\gamma^2)} - \frac{\{(B_\gamma^2 + C_\gamma^2)R_S^2 - [C_\gamma(y_D-y_0)^2 + B_\gamma(z_D-z_0)^2]\}^{\frac{1}{2}}}{(B_\gamma^2 + C_\gamma^2)}, \] (23)
\[ B_\gamma = (y_1-y_D)/d_\gamma, \] (24)
\[ C_\gamma = (z_1-z_D)/d_\gamma, \] (25)

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Let
\[ n_n = \text{neutron absorption factor}, \]
\[ n_\gamma = \text{gamma-ray absorption factor}, \]
then
\[ n_n = \exp \left( -\sum \delta_n \right), \tag{26} \]
\[ n_\gamma = \exp \left( -\sum \delta_\gamma \right). \tag{27} \]

If
\[ e_{\text{DET}} = \text{efficiency of the gamma-ray detector,} \]
defined as the ratio of detected (in the full-energy peak) to incident gamma rays,
then
\[ Y_0 = \sum_{\alpha,\beta,i,j,k} \frac{d^3n}{d\Omega \cdot d\Omega} N_s \frac{\sigma(d\sigma/d\Omega)}{N_s} e_{\text{DET}} \cdot \]
\[ \tag{28} \]

\( Y_0 \) is a function of \( E_n \) and \( \Theta_{n\gamma} \). The energy \( E_n \) lies in the range \((0, E_{n,max})\) while the scattering angle \( \Theta_{n\gamma} \) is in the range \((0,\pi)\). Therefore, it is possible to define resolution functions \( \delta(E_n) \) and \( \mathcal{A}(\Theta_{n\gamma}) \) such that
\[ Y_0 = \int_0^{E_{n,max}} \delta(E_n) dE_n, \tag{29} \]
\[ Y_0 = \int_0^\pi \mathcal{A}(\Theta_{n\gamma}) d\Theta_{n\gamma}. \tag{30} \]

These resolution functions can be used to compute the average neutron energy \( \langle E_n \rangle \) and the average scattering angle \( \langle \Theta_{n\gamma} \rangle \) according to the formulas
\[ \langle E_n \rangle = \frac{1}{Y_0} \int_0^{E_{n,max}} E_n \delta(E_n) dE_n, \tag{31} \]
\[ \langle \Theta_{n\gamma} \rangle = \frac{1}{Y_0} \int_0^\pi \Theta_{n\gamma} \mathcal{A}(\Theta_{n\gamma}) d\Theta_{n\gamma}. \tag{32} \]

The relationship between \( Y_0 \) and the differential cross section \((d\sigma/d\Omega)\) is indicated in Eq. (28). Let
\[ \langle \frac{d\sigma}{d\Omega} \rangle_\gamma = \text{value of the differential cross section corresponding to neutron energy } \langle E_n \rangle \text{ and scattering angle } \langle \Theta_{n\gamma} \rangle, \]

and define
\[ \xi_\gamma = \frac{(d\sigma/d\Omega)_\gamma}{\langle \frac{d\sigma}{d\Omega} \rangle_\gamma}, \quad (33) \]

then Eq. (28) can be rewritten in the form
\[ Y_0 = \langle \frac{d\sigma}{d\Omega} \rangle_\gamma \sum_{\alpha, \beta, i, j, k} \frac{n_T}{n_T} \left( \frac{n_R}{d_R} \right) \left( \frac{\xi_\gamma}{d_\gamma} \right) N_{\xi_\gamma} V_j \xi_\text{DET} \]

which explicitly relates the gamma-ray yield to the differential cross section for a specific neutron energy and scattering angle. This formalism requires an approximate knowledge of \( \xi_\gamma \) (the shape of the differential cross section function in terms of neutron energy and scattering angle). In practice, most of the contributions to \( Y_0 \) come from limited ranges of neutron energy and scattering angle. Therefore, one estimates the behavior of \( \xi_\gamma \) for the regions of interest and applies this estimate in computations. Improved accuracy can be achieved by the process of iteration. Experience has shown no more than two passes are required for most applications.

3.2 Results of Numerical Studies

It is worthwhile to factor the gross solid-angle and sample-volume dependence from the expressions for \( Y_0 \). This can be achieved through definition of the quantity \( \overline{Y}_0 \) as follows:
\[ Y_0 \equiv (\pi R_S^2 H D_n D_\gamma^2) \overline{Y}_0. \quad (35) \]

In this section, sample geometry and absorption effects are investigated solely in terms of the behavior of \( \overline{Y}_0 \).
A Systems Engineering Laboratories Model 840 MP digital computer was utilized in computation of $\bar{Y}_0$ for various experimental conditions. A nominal parameter set for these calculations appears in Table I. The variation of $\bar{Y}_0$ in response to departures from the conditions represented by these parameters is investigated in the present section.

Selection of an appropriate set of mesh parameters $n_T$, $n_H$, $n_R$ and $n_\phi$ is an important consideration. Coarse meshes lengthen the computation time unnecessarily. The computations were least sensitive to $n_T$ and most sensitive to $n_R$ as expected. The mesh parameters listed in Table I appear to be satisfactory for most practical applications.

The effects of radiation absorption were investigated by computing $\bar{Y}_0$ for four sets of parameters which differ from each other only in the assumed values for $\Sigma_{nt}$ and $\Sigma_{\gamma T}$: i) neutron and gamma-ray absorption (Table I), ii) gamma-ray absorption only ($\Sigma_{nt} = 0$), iii) neutron absorption only ($\Sigma_{\gamma T} = 0$), and iv) no absorption ($\Sigma_{nt} = \Sigma_{\gamma T} = 0$). The effects of absorption are a reduction of gamma-ray yield and a distortion of the observed angular distribution. The relative gamma-ray yields at $\Theta_{\Delta ET} = 90^\circ$ for these four cases are: i) 0.34, ii) 0.50, iii) 0.68, and iv) 1.00. The induced anisotropy for each situation is shown in Fig. 3. These distortions appear to be well represented by the expression

$$\bar{Y}_0 \left( \Theta_{\Delta ET} \right) \propto \bar{Y}_0 \left( 90^\circ \right) \left( 1 - \Delta_0 \cos \Theta_{\Delta ET} \right) \quad (36)$$

with $\Delta_0$ assuming the following positive values for the four cases considered: i) 0.172, ii) 0.0658, iii) 0.0056,
and iv) 0.0021. Geometric effects and neutron attenuation alone produce very little distortion; however, gamma-ray absorption (particularly in combination with neutron absorption) skews the observed angular distribution about \( \Theta_{\text{DET}} = 90^\circ \) so that the back-angle yield exceeds the forward-angle yield.

The relative contributions to \( \bar{V}_O \) for \( \Theta_{\text{DET}} = 90^\circ \) from the midplane sample elements are presented for each of the four cases considered as follows: i) Fig. 4, ii) Fig. 5, iii) Fig. 6, and iv) Fig. 7. In conjunction with the results presented in Figs. 4-7, it is worthwhile to consider the ratios of yields from larger segments of the sample. Define the "back" of the sample as that half of the cylinder which is farthest from the neutron source, and the "front" of the sample as the opposite half. Similarly, label as "far" the sample half farthest from the gamma-ray detector. The opposite half is labelled as "near". Computed "front"-to-"back" and "near"-to-"far" ratios are listed in Table II.

The parameter \( D_n \) was varied over the range 5-50 cm with other parameters fixed at the values given in Table I. The value of \( \bar{V}_O \) for \( \Theta_{\text{DET}} = 90^\circ \) increased by only 3.3% as \( D_n \) increased from 5 to 50 cm. This would seem to imply that a parallel neutron beam approximation is warranted. However, variation in the distortion of the angular distribution was more pronounced as \( \Delta_0 \) decreased from 0.268 for \( D_n = 5 \) cm to 0.121 for \( D_n = 50 \) cm. For small values of distance \( D_n \), the sample subtends a sizeable solid angle so that the average scattering angle becomes quite sensitive to \( D_n \). Therefore, use of the parallel neutron beam approximation is not recommended.

The parameter \( D_Y \) was varied over the range 30-200 cm with other parameters fixed at the values given in Table I. The value of \( \bar{V}_O \) for \( \Theta_{\text{DET}} = 90^\circ \) decreased by only 1.2%
as $p_\gamma$ increased from 30 to 200 cm, and the distortion parameter $\Delta_0$ decreased from 0.190 to 0.168. A parallel gamma ray approximation could be justified for these calculations.

Variation of the sample height $H$ produces very little effect on $\overline{V}_0$; however, the gamma-ray yield and distortion of the angular distribution depend critically upon the sample radius $R_S$. Computations were made for $R_S = 0.635, 0.95, 1.27, 1.59, 1.905$ and $2.54$ cm with the other parameters fixed at the values listed in Table I. The results of these calculations appear in Table III. The near constancy of $R_S \overline{V}_0$ for large values of $R_S$ implies that, in this domain, the total gamma-ray yield from the sample increases more or less linearly with sample radius rather than as the square of the radius. The difference is due to absorption. Clearly, the total gamma ray yield from the sample also increases linearly with sample height. A set of calculations was made assuming no neutron or gamma-ray absorption and using values of $H = 2R_S$ ("square" samples). As $R_S$ increases from 0.635 cm to 2.54 cm, $\overline{V}_0$ decreases by 1.7%. This result indicates that the yield per unit volume depends only slightly on the size of the sample in the absence of absorption.

The most significant geometric effect is the loss of angular resolution which results from the use of large samples. The maximum geometric angular range for $\Theta_{nY}$ associated with the conditions of Table I is $\sim 19^\circ$. The contribution to this spread from the neutrons is $\sim 17^\circ$ while that from the gamma rays is $\sim 2^\circ$. The resolution functions $A(\Theta_{nY})$, as defined by Eq. (30), were determined for several values of $\Theta_{DET}$ using Table I parameters. The results are presented in Fig. 8. Values of $\langle \Theta_{nY} \rangle$, computed with these resolution functions, differ significantly from the corresponding detector angles $\Theta_{DET}$ when $\Theta_{DET}$ approaches
0° or 180°. This result illustrates the well-known fact that relatively small samples are required for the measurement of differential cross sections near 0° or 180°.

Gamma-ray angular distributions for (n,γ) reactions are symmetric about θ\textsubscript{ny} = 90°. Computations were made using various assumed gamma-ray differential cross section functions \((d\sigma/dΩ)γ\). These calculations indicate that, for positive values of \(Δ_0\), the formula

\[
\overline{Y}_0 \sim \text{(Constant)} \left(1 - Δ_0 \cos θ_{DET}\right) \langle(d\sigma/dΩ)γ\rangle
\]

(37)

is valid to a considerable degree of accuracy. Furthermore, the distortion parameter \(Δ_0\) and the average scattering angle \\langleθ\textsubscript{ny}\rangle\ are very insensitive to the shape of the differential cross section \((d\sigma/dΩ)γ\). Eq. (37) resembles Eq. (34) and it is concluded that the complicated sum given in Eq. (34) has a simple angular dependence. This particular result will be designated the "factorization rule" since it provides a prescription for relating the observed gamma-ray yield to the shape of the differential cross section. Fig. 9 demonstrates the factorization rule. This rule applies reasonably well for most realistic applications even when multiple scattering is taken into consideration (Section 4.2). Application of the factorization rule leads to a significant labor reduction in processing angular distribution data.

Realistically, the neutron field produced by proton bombardment of natural lithium is a mixture of first- and second-group neutrons plus some breakup neutrons at higher bombarding energies (Section 1). The energies and angular distributions of these components differ and this will influence the overall gamma-ray and monitor yields observed. Of concern, however, is the effect of superposition of contributions on the applicability of the factorization rule. To investigate this point, computations were made for several incident energies assuming realistic lithium first-
and second-group neutrons plus some breakup neutrons at higher bombarding energies (Section 1). The energies and angular distributions of these components differ and this will influence the overall gamma-ray and monitor yields observed. Of concern, however, is the effect of superposition of contributions on the applicability of the factorization rule. To investigate this point, computations were made for several incident energies assuming realistic lithium first- and second-group neutron sources as well as the hypothetical isotropic source identified in Table I. These calculations show that the distortion parameter $\Delta$ is relatively insensitive to the shape of the neutron-source reaction angular distribution. The parameter $\Delta$ varies with neutron energy since it depends on the total cross section. However, since $\Delta \ll 1$ for typical conditions, the energy dependence of $\Delta$ does not affect the angular distributions severely. For example, $\Delta$ is found to vary by $\sim 30\%$ over the range $E_n = 0.9-2$ MeV for the sample described in Table I; however, the ratio $\overline{Y}_0(90^\circ)/\overline{Y}_0(90^\circ)$ varies by only $\sim 3\%$ over this range. Therefore, it is often possible to apply the factorization rule for multigroup neutron sources. The formulas suggested by the results of numerical analysis are

$$\overline{Y}_0(\text{multigroup}) \lesssim \text{(Constant)} \ (1 - \langle \Delta \rangle \cos \Theta_{\text{DET}}) \ .$$

$$S_m \ G_m \ \langle (d\sigma/d\Omega) \rangle_m \ , \quad (38)$$

$$\langle \Delta \rangle = \left( S_m \ G_m \ \Delta_m / S_m \ G_m \right) \quad (39)$$

The parameters $G_m$ depend upon the group intensities and sample absorption properties for neutrons in these groups.

The factorization rule is not a rigorous product of the formalism, but is an empirical concept which has been
distilled from the results of numerical calculations. Caution should be exercised when using this rule in processing data, particularly for measurements involving multigroup neutron sources.

4. EFFECTS OF NEUTRON MULTIPLE SCATTERING

The obvious result of multiple scattering is the enhancement of the observed yield relative to that predicted by the computations of the previous section. The objectives of the present section are to describe the method used for the evaluation of the multiple scattering parameter $\alpha_{\text{TOT}}$ and to investigate the dependence of multiple scattering on various experimental factors.

4.1 Mathematical Formalism

The approach taken in this work is to calculate values of $Y_\lambda$ for $\lambda = 0, 1, \ldots, k$ (see Section 1) by statistical methods and then compute the partial multiple-scattering parameters $\alpha_\lambda$ by means of Eq. (2). The total multiple-scattering parameter $\alpha_{\text{TOT}}$ is given by Eq. (3). There are several ways to formulate Monte-Carlo problems [13]. The present approach generally resembles that which is employed in multi-dimensional Monte-Carlo integration. The fundamental assumption of Monte-Carlo integration is that

$$\int_V dq \cdot f(q) \overset{\sim}{\sim} \frac{1}{N_{\text{hist}}} \sum_{i=1}^{N_{\text{hist}}} S_i f(q_i)$$

(40)

for a sufficiently large number of histories $N_{\text{hist}}$, where $q_i$ is selected at random from a region of $v$-dimensional Cartesian space defined by

$$q = (q_1, q_2, \ldots, q_v)$$

(41)

$$dq = (dq_1, dq_2, \ldots, dq_v)$$

(42)

$$V = (q_{1,\text{max}} - q_{1,\text{min}}) (q_{2,\text{max}} - q_{2,\text{min}}) \ldots \ldots (q_{v,\text{max}} - q_{v,\text{min}}).$$

(43)
The region of space defined by the volume $V$ is a $v$-dimensional rectangle and is selected so that the true region of interest lies entirely within $V$. The Monte-Carlo process consists of selecting points $q_1$ at random in the larger region. If $q_1$ falls outside the true region of interest, then $f(q_1) = 0$ (a "miss"). This approach does not lead to optimum efficiency, however such wastefulness is usually tolerable with high-speed digital computers and avoids many computational complexities.

Some of the variables used in this analysis are defined in Sections 1 and 3; others are defined at appropriate points in the present section.

First, computation of $Y_0$ by Monte-Carlo methods is considered (see Fig. 10). The origin of coordinates is the neutron source (assumed here to be a point). The beam line is the $z$-axis, the sample axis is normal to the $y-z$ plane as in Section 3. The center of the sample is at the coordinates $(0,0,D_n)$. The gamma-ray detector is a point in the $y-z$ plane located a distance $D_\gamma$ from the center of the sample. The first-scattering point $S_1$ in the sample is identified by the vector $\hat{x}_1$ with coordinates $(x_1,y_1,z_1)$. A related spherical coordinate system can be defined by the equations

\begin{align}
    x_1 &= r_1 \sin \theta_1 \cos \phi_1, \\
    y_1 &= r_1 \sin \theta_1 \sin \phi_1, \\
    z_1 &= r_1 \cos \theta_1.
\end{align}

The sample is located entirely within a region of space defined by the following expressions

\begin{align}
    r_1,\min &\leq r_1 \leq r_1,\max, \\
    r_1,\min &= D_n - R_S, \\
    r_1,\max &= [(D_n + R_S)^2 + \frac{1}{4} H^2 + R_S^2],
\end{align}

\[-21-\]
\[ 0 < \phi_1 < 2\pi, \quad (50) \]
\[ 0 < \theta_1 < \theta_{1,\text{max}}, \quad (51) \]
\[ \theta_{1,\text{max}} = \tan^{-1} \left[ \frac{(\frac{1}{2} H^2 + R_S^2)^{\frac{1}{2}}}{R_n - R_S} \right]. \quad (52) \]

The energy and angular dependence of all physical parameters is taken into consideration as well as kinematic effects. The gamma-ray yield from \((n,\gamma)\) reactions initiated by unscattered neutrons is given by

\[ Y_0 \sim \frac{V_1}{N_{\text{hist}}} \sum_{i=1}^{N_{\text{hist}}} \Gamma \quad (53) \]

with

\[ \Gamma_0 = \begin{cases} 
\Gamma \quad \text{if } \hat{x}_1 \text{ is inside the sample,} \\
0 \quad \text{otherwise (a "miss"),}
\end{cases} \quad (54) \]

\[ \eta_1 = \exp(-\delta n nT), \quad (55) \]

\[ V_1 = 2\pi (r_{1,\text{max}} - r_{1,\text{min}}) \theta_{1,\text{max}}. \quad (56) \]

For \( \hat{x}_1 \) to lie inside the sample, it is required that

\[ -H/2 < x_1 < H/2, \quad (57) \]

\[ (\frac{y_1}{2} + (z_1 - D_n)^2)^{\frac{1}{2}} < R_S. \quad (58) \]

Although determination of \( Y_0 \) by Monte-Carlo integration is technically simpler than the approach described in Section 3, it is slower since quite a few histories are required for convergence. Furthermore, it is not possible to derive the midplane-yield profile information presented in Figs. 4-7 from a simple Monte-Carlo treatment. However, for \( \ell > 1 \) it is impractical to determine \( Y_\ell \) by any method other than Monte-Carlo analysis.
Next, consider computation of $Y_1$ (see Fig. 10). The scattering preceding the (n,\textit{n}γ) event can be either elastic or inelastic. The possibility for more than one neutron channel adds an additional complication to the computations. Some neutrons which scatter in the vicinity of point $S_1$ propagate toward point $S_2$ defined by the vector $x_2$. Assume that there are $N_1$ distinct scattering processes applicable to the first scattering point $S_1$. The j-th process is defined by the parameters $A_{1j}$, $Q_{1j}$ and $(d\Sigma/d\Omega)_{1j}$ where

\begin{align*}
A_{1j} &= \text{mass of the target nucleus}, \\
Q_{1j} &= \text{reaction Q-value}, \\
(d\Sigma/d\Omega)_{1j} &= \text{macroscopic differential neutron-scattering cross section}.
\end{align*}

For simplicity, the inelastic scattering processes are assumed to be isotropic since they are nearly so in reality. Let $\psi_1$ be the total macroscopic scattering cross section defined by the equation

$$
\psi_1 = \frac{1}{S_{j=1}^{N_1}} (d\Sigma/d\Omega)_{1j}.
$$

then the relative probability $P_{1j}$ of each process is given by the equation

$$
P_{1j} = \frac{(d\Sigma/d\Omega)_{1j}}{\psi_1}.
$$

Since

$$
\frac{1}{S_{j=1}^{N_1}} P_{1j} = 1,
$$

the unit interval can be divided by a set of $N_1$ points \{$\rho_{1j}$\} defined by the equation

$$
\rho_{1j} = \frac{1}{S_{l=1}^{j}} P_{1l}.
$$
A random number \( R \) is selected for each history. The neutron is then assumed to propagate from point \( S_1 \) to point \( S_2 \) by means of the \( j \)-th scattering process if

\[
\rho_{1,j-1} R \leq \rho_{1,j}.
\]

(63)

Although the particular scattering process is selected by random sampling in the space of open channels, the macroscopic differential scattering cross section used in the computation is \( \psi_1 \).

A new spherical coordinate system with origin at \( S_1 \) is defined. The cartesian coordinates (origin at the neutron source) and spherical coordinates in the new system for point \( S_2 \) are related by the equations

\[
x_2 = x_1 + r_2 \sin \theta_2 \cos \phi_2,
\]

(64)

\[
y_2 = y_1 + r_2 \sin \theta_2 \sin \phi_2,
\]

(65)

\[
z_2 = z_1 + r_2 \cos \theta_2.
\]

(66)

The region of space defined by the expressions

\[
0 \leq \theta_2 \leq \pi,
\]

(67)

\[
0 \leq \phi_2 \leq 2\pi,
\]

(68)

\[
0 \leq r_2 \leq r_{2,\text{max}},
\]

(69)

\[
r_{2,\text{max}} = (H^2 + 8 R_S^2)^{\frac{1}{2}},
\]

(70)

encompasses the entire sample. Then,

\[
Y_1 \approx \frac{V_1 V_2}{N_{\text{hist}}} \sum_{i=1}^{N_{\text{hist}}} \Gamma_{1i}
\]

(71)

with

\[
\Gamma_1 = \begin{cases} 
F_n \eta_1 \sin \theta_1 \psi_1 \eta_2 \sin \theta_2, \\
N_S (d\sigma/d\Omega)_\gamma \left( \frac{\eta}{d\gamma} \right) \varepsilon_{\text{DET}} \\
0 \text{ otherwise (a "miss"),}
\end{cases}
\]

(72)

\[
\eta_2 = \exp(-r_2 \frac{F_n}{\varepsilon_{\text{fT}}}),
\]

\[
V_2 = 2\pi^2 r_{2,\text{max}}^2.
\]
For \( \hat{x}_2 \) to lie inside the cylinder, it is required that

\[
- \frac{H}{2} < x_2 < \frac{H}{2},
\]

\[
[y_2^2 + (z_2 - D_n)^2]^{1/2} < R_S.
\]

(73)

(74)

Generalization to arbitrary orders of multiple scattering is straightforward. The expression for \( Y_k \) is

\[
Y_k \sim \frac{V_1 V_2 \ldots V_{k+1}}{N_{\text{hist}}} \sum_{i=1}^n r_{ki},
\]

(75)

with

\[
\Gamma_k = \left\{ \begin{array}{l}
F_n (\pi \eta_k \sin \theta_k \psi_k) \eta_{k+1} \sin \theta_{k+1} \\
N_S (d\sigma/d\Omega) \gamma \left( \frac{\eta_k}{\eta_2} \right) e_{\text{DET}} \\
\text{if } \hat{x}_1, \ldots, \hat{x}_{k+1} \text{ are inside the sample}, \\
0 \text{ otherwise (a "miss")},
\end{array} \right.
\]

(76)

The parameters required for computation of all the higher-order scattering contributions resemble those described for computation of \( Y_1 \).

Values of \( \Gamma_{0i}, \ldots, \Gamma_{ki} \) are computed for every history \((i=1, \ldots, N_{\text{hist}})\). Whenever a particular \( \Gamma_{k+1} = 0 \) (a "miss"), the higher-order expressions \( \Gamma_{k+1, i}, \ldots, \Gamma_{ki} \) are automatically equal to zero too. Thus, the efficiency for computation of \( Y_k \) declines with increased scattering order \( k \). However,

\[
Y_0 >> Y_1 >> \ldots >> Y_k,
\]

(77)

so it is unnecessary to determine the high-order contributions to \( Y_{\text{TOT}} \) (Eq. 1) as accurately as the low-order contributions.

4.2 Results of Numerical Studies

Multiple-scattering calculations were performed with the SEL 84OMP computer using a code named GAMSCT. This
code has been written in FORTRAN IV and a listing of the orders is given in Appendix B.

A nominal set of parameters, which provided a starting point for numerical studies of multiple scattering, is given in Table IV. Experience indicates that for $k=3$, satisfactory accuracy in computation of $\alpha_{TOT}$ is achieved for $N_{hist} \approx 100,000$. This value was selected for all calculations. Typical efficiencies ('hit' percentages) of the Monte-Carlo trials are as follows: $Y_0(54.9\%)$, $Y_1(14.2\%)$, $Y_2(3.7\%)$ and $Y_3(0.8\%)$. Relative values of $Y_0$, $Y_1$, $Y_2$ and $Y_3$ for various $\Theta_{DET}$ are plotted in Fig. 11. It is seen that the ratio $Y_{l+1}/Y_l$ is more or less independent of $l$, and furthermore for all $l$,

$$Y_l(\Theta_{DET}) \approx Y_l(90^\circ) (1 - \Delta_l \cos \Theta_{DET}), \quad (78)$$

$$\Delta_0 > \Delta_1 > \Delta_2 > \Delta_3 \approx 0. \quad (79)$$

Intuitively, one expects the effects of geometric anisotropy to be washed out by multiple scattering. Eq. (79) supports this contention. Eq. (78) indicates that the factorization rule applies for an assumed isotropic differential cross section $(d\sigma/d\Omega)_Y$. Actually it also applies reasonably well for most realistic differential cross section functions. However, the factorization rule does fail in extreme cases where $(d\sigma/d\Omega)_Y$ approaches zero for $\Theta_{nY} = 0^\circ$ or $180^\circ$. Under these conditions, the observed yield for $\Theta_{DET}$ near $0^\circ$ or $180^\circ$ is dominated multiple scattering in a fashion which cannot be explained by a simple rule.

The effect of sample size was tested by varying $R_S$ and $H (H = 2R_S)$ with respect to the values in Table IV. The results are presented in Fig. 12. The contributions from second- and higher-order scattering are negligible for small samples. Variation of $D_n$ and $D_Y$ over realistic ranges produced very little effect on the computed multiple-scattering parameters.
Computations performed with various assumed realistic neutron-source reactions indicate that the multiple-scattering correction parameters are relatively insensitive to the properties of the neutron source. Therefore, it is reasonable to compute $\alpha_{\text{TOT}}$ for various energies $E_n$ and angles $\theta_{\text{DET}}$ assuming an isotropic, monoenergetic neutron source.

The multiple-scattering parameters are relatively insensitive to $\Sigma_{\text{nT}}$ and $\Sigma_{\text{yT}}$. However, they depend critically on the magnitudes of the scattering cross sections. The relationship

$$\alpha_{\lambda} \sim (\text{Constant}) \Sigma_{\text{EL}} $$

(80)

gives a rough indication of this dependence for the simple case of energy-independent elastic scattering. The multiple scattering parameters are considerably less sensitive to the shape of the neutron scattering angular distributions.

5. COMPARISON OF EXPERIMENTAL AND COMPUTED RESULTS FOR NATURAL IRON SAMPLES

Measurements were performed with seven natural iron samples to test the validity of the methods described in this report. The sizes of the samples investigated were $R_S = 0.635, 0.953, 1.27, 1.59, 1.91, 2.22$ and $2.54$ cm ($H = 2 R_S$). Realistic energy-averaged cross sections were utilized in the computations (Section 2 and Refs. 14 and 15). The facility described in Ref. 1 was utilized for the irradiations. A 0.1-MeV-thick natural lithium target was bombarded with 3.68-MeV protons. Approximately 90% of the neutrons originated from the $^7\text{Li}(p,n)^7\text{Be}$ reaction ($E_n \sim 2$ MeV) and 10% came from the $^7\text{Li}(p,n)^7\text{Be}^*$ reaction ($E_n \sim 1.535$ MeV); the proton energy was slightly
below threshold for the $^7\text{Li}(p,n)^3\text{He}^4\text{He}$ breakup reaction. The Ge(Li) detector was situated at $\theta_{\text{DET}} = 90^\circ$. The relative neutron fluence was monitored by time-of-flight techniques using a plastic scintillator.

The full-energy peak yields for the 0.846-MeV line from the $^{56}\text{Fe}(n,n^{'}\gamma)^{56}\text{Fe}$ reaction were divided by the masses of the corresponding samples (proportional to the volume) to determine quantities proportional to the yield per atom. All measurements were performed in identical geometry, so the yields per atom deduced are proportional to $\overline{Y}_0 (1 + \alpha_{\text{TOT}})$ as defined in Sections 1-4.

Measurements for samples with $R_S < 0.635$ cm were not practical because of background problems; however, computations were performed for a wide range of sample sizes including $R_S$ near zero. In the limit of very small samples, geometry, absorption and multiple-scattering effects vanish. The experimental and computed values were normalized so that the yield per atom approaches unity for very small samples.

Four sets of computations were performed. The assumptions made in these calculations are as follows: i) no absorption, geometric corrections only, ii) absorption of neutrons and gamma-rays with the neutron total cross section used for absorption calculations and multiple scattering neglected, iii) identical to (ii) except that the total non-elastic cross section is used for the neutron absorption calculations, and iv) identical to (ii) except multiple scattering is considered (most realistic approach). The results of these calculations are compared with the experimental data in Fig. 13. The calculations labelled (iii) and (iv) both agree well with the experimental results; the agreement of set (iv) values is superior as anticipated. The agreement for such a wide range of sample sizes is very encouraging.
(the diameter of the largest sample considered is equivalent to 1.4 mean free path lengths for neutrons and 2.5 mean free path lengths for 0.846-MeV photons).

The assumptions made for the set (iii) calculations were suggested by Day [3]. The Day approximation is widely used by researchers in the analysis of (n,XY) data. It is appealing because it eliminates the necessity for performing multiple-scattering calculations. The results shown in Fig. 13 correspond to $\theta_{DET} = 90^\circ$.

Computations were performed to compare the predictions of the Day approximation with those from the more realistic treatment at other angles. The results of this analysis show that these two approaches yield results which agree within $\sim 3\%$ for $\theta_{DET} = 90^\circ - 150^\circ$; however, the agreement for $\theta_{DET} = 30^\circ - 90^\circ$ is only within $\sim 7\%$ (for an iron sample with $R_S = 1.9$ cm). This suggests that for careful work, where accuracies of better than 10% are sought, it is advisable to employ a realistic treatment which includes multiple-scattering analysis. In applications where such accuracy is not sought, or is unfeasible, the Day approximation appears to be warranted since it saves considerable labor.

6. CONCLUSIONS

The formalism described in this report provides a means for determining differential cross section data for (n,XY) reactions from measurements made using relatively large cylindrical samples provided that accurate absorption and scattering cross section data is available.

Geometric effects reduce angular resolution and make it difficult to measure differential cross sections near 0° or 180°; otherwise, they have a relatively minor influence on the measurements.

The absorption of radiation reduces the overall yield and distorts angular distributions. Under most conditions, this distortion assumes the form $1 - \Delta \cos^{\theta}_{DET}$.
$(\Delta \rightarrow 0)$; the shape of the differential cross section can be deduced from the experimental data by factoring out this simple angular dependence. Factorization is possible, even in the presence of multiple scattering, for most realistic situations and this saves considerable labor in processing data.

The Day approximation [3] permits one to avoid making detailed multiple scattering calculations, and appears to be an acceptable approach when accuracies of no better than $\sim 10\%$ are acceptable.
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APPENDIX A

COHERENT PHOTON SCATTERING

Photons which scatter coherently in the sample are indistinguishable from those which suffer no interaction. The coherent scattering cross sections depend upon photon energy $E_\gamma$ and atomic number $Z$. They decrease rapidly with $E_\gamma$. For this reason, the coherent scattering corrections are relatively small for most materials of interest when $E_\gamma$ exceeds a few hundred kilovolts.

In order to estimate the correction, a model which assumes a well-defined, distributed gamma-ray source intensity in a solid cylinder is employed. The unscattered and coherently-scattered photon yield at a distant detector is computed as described below.

If $\Sigma_{Y,COH}$ is the integrated macroscopic coherent scattering cross section, then the macroscopic differential scattering cross section is given by

$$ (d\sigma/d\Omega)_{Y,COH} = \frac{3\Sigma_{Y,COH}}{16\pi} (1 + \cos^2 \theta_\gamma). \quad (81) $$

Assume that the cylinder is divided into a large number of discrete elements. Let

$\mathbf{x}_i = \text{coordinates of the center of the } i\text{-th element}$

$\Delta V_i = \text{isotropic gamma-ray source strength density for the } i\text{th component (gamma rays/sr/cm}^3\text{),}$

$\mathbf{x}_\Delta = \text{coordinates of the detector.}$

$\delta_i = \text{distance through sample material which the photon must penetrate to reach the detector}$

$\text{if it originates at point } \mathbf{x}_i. \text{ (See Section 3),}$

$Y_U = \text{yield at the detector due to unscattered photons}$

$Y_S = \text{yield due to photons which have scattered once coherently in the sample.}$
\[ Y_U \cong S \frac{S_i \Delta V_i \exp(-\sum_{i} \gamma_i \delta_i)}{|\hat{x}_{0i} - \hat{x}_D|} \epsilon_{\text{DET}}, \quad (82) \]

\[ Y_S \cong S \sum_{i,j} S_i \Delta V_i \Delta V_j \exp\left[-\sum_{i,j} (\delta_i + |\hat{x}_i - \hat{x}_j|)\right] \cdot \frac{|\hat{x}_i - \hat{x}_j|^2 |\hat{x}_j - \hat{x}_D|^2}{|\hat{x}_i - \hat{x}_j|^2 |\hat{x}_j - \hat{x}_D|^2} \cdot (d\Sigma/d\Omega)_{\gamma,\text{COH},ij} \epsilon_{\text{DET}} \quad (83) \]

The problem has been formulated in cylindrical coordinates and a code is available for operation on the SEL 840 MP digital computer. A number of calculations were performed assuming a constant value for \( S_i \) throughout the sample. Samples with \( R_S = 1.9 \text{ cm} \) and \( H = 3.8 \text{ cm} \), fabricated from Li, Al, Ti, Fe, Zn and Mo, were considered. The detector was assumed to be 130 cm from the sample. The ratio \( Y_U/(Y_U+Y_S) \) was computed for \( E_\gamma = 0.1, 0.3, 0.5, 0.8, 1.0, 2.0 \) and 5.0 for these samples. The results appear in Table V. The coherent scattering correction is clearly quite small if not negligible for most cases of interest.
APPENDIX B

LISTING OF CODE GAMSCT

Code GAMSCT was developed to perform the multiple-scattering computations discussed in Section 4 of this report. This code is written in ASI Standard FORTRAN IV. All code input is from cards (Unit 4). Output is produced on a teletype (Unit 1) and a line printer (Unit 5). The version of this code listed here is operated on a Systems Engineering Laboratories Model 840 MP digital computer.
GMSCT=D.L.SMITH=SEL 840MP

DIMENSION ENT(25),SIGN(25),EGP(25),SIGGP(25),NWGP(6),EWGP(6,15),WGMSCT
1G(6,15),QNS(6),A2NS(6),NNS(6),ENS(6,25),SIGNS(6,25),MNNS(6),WNNS(GMSCT
26,10),EWNS(6,10,15),WNNS(6,10,15),DSIGNS(6),WT(10),YLD(5),YI,DSIHM(5)GMSCT
3,PSI(6),INDEX(4),JNDFX(5),NHIT(5),NLEV(6),A(25),R(25),W0RK(25)

DATA PI/3.14159/

VALUE(V,VMIN,VMAX)=VMIN*V*(VMAX-VMIN)
SEPAR(X1,Y1,Z1,X2,Y2,Z2)=SQRT((X1-X2)*(X1-X2)+(Y1-Y2)*(Y1-Y2)+(Z1-Z2))

C C
CONTROl

1 READ(4,2) IC
2 FORMAT(I1)
G0 TO(10,20,30,50),IC
10 PAUSE
G0 TO 1

C C
READ INTERPOLATION TABLES

C C
20 READ(4,21) MNT
21 FORMAT(1615)
READ(4,22) (ENT(I),SIGN(I),I=1,MNT)
22 FORMAT(8E10.4)
READ(4,21) MGP
READ(4,22) (EGP(I),SIGGP(I),I=1,MGP)
READ(4,21) MWGP
IF(MWGP) 23,25,23
23 DO 24 I=1,MWGP
READ(4,21) NWGP(I)
M=NWGP(I)
24 READ(4,22) (EWGP(I,J),WGP(I,J),J=1,M)
25 READ(4,21) MNS
DO 28 I=1,MNS
READ(4,22) QNS(I),A2NS(I)
READ(4,21) NNS(I)
M=NNS(I)
READ(4,22) (ENS(I,J),SIGNS(I,J),J=1,M)
READ(4,21) MNNS(I)
IF(MNNS(I)) 26,28,26
26 L=MNNS(I)
DO 27 J=1,L
READ(4,21) NWNS(I,J)
M=NWNS(I,J)
27 READ(4,22) (EWNS(I,J,K),WNNS(I,J,K),K=1,M)
28 CONTINUE

C C
READ AND WRITE BASIC PARAMETERS

30 READ(4,31) NSCAT,NHIST
31 FORMAT(I1,16)
READ(4,22) RS,H,DNO,GO
READ(4,22) EG,SIGGG,ENTHG
READ(4,22) A1T,AT,QT
READ(4,32) ET,NNT
32 FORMAT(E10.4,15)
IF(NWT) 34,34,33
33 READ(4,22) (WT(I),I=1,NWT)
34 WRITE(5,35) NSCAT,NHIST
35 FORMAT(1H1/11HNSCAT,NHIST/I1,16)
36 WRITE(5,36) RS,H,DNO,DGO
36 FORMAT(1H1/12HRS,H,DNO,DGO/4E10.4)
37 WRITE(5,37) EG,SIGT,ENTHG
38 WRITE(5,38) A1T,A2T,QT
38 FORMAT(10H1T,A2T,QT/3E10.4)
39 IF(NWT) 40,42,40
40 WRITE(5,41)
41 FORMAT(5HWT(I))
42 WRITE(5,22) (WT(I),I=1,NWT)
43 FORMAT(/5H00.

READ AND WRITE SCATTERING ANGLE, CONVERT TO RADIANS

50 READ(4,22) THTANK
51 WRITE(5,51) THTANK
51 FORMAT(7HTHTANK=E10.4)
52 THDET=PI*THTANK/180.0

PRELIMINARY CALCULATIONS

RSRS=RS*RS
HH=H*H
HD2=0.5*H
RMIN=DNO*RS
RMAX=SQRT(DNO*DSQRT(2.0*DSQRT(RMIN)*RSRS+0.25*HH)
TH1MAX=ATAN(SQRT(0.25*HH+RSRS)/RMIN)
VOL1=2.0*PI*(RMAX-RMIN)*TH1MAX
VOLH=2.0*PI*PI*RMAX
YS=DGC*THDET
ZD=DNO*DGO*THDET
D0 60 I=1,NSCAT
NHIT(I)=0
60 YLDSUM(I)=0.0
61 D0 61 I=1,MNS
61 NLEV(I)=0
NHIST=1

START OF HISTORY LOOP

C

100 D0 101 I=1,NSCAT
101 JINDEX(I)=0

SELECT SCATTERING POINT S(I)

R=RANF(-1)
R1=RANF(I)
R1=RANF(-1)
TH=VALUE(R,0.0,TH1MAX)
R1=RANF(-1)
PHI=VALUE(R,0.0,PI1)
SINTH = SIN(TH)
X = RR * SINH * COS(PHI)
Y = RR * SINH * SIN(PHI)
Z = RR * COS(TH)

IF (X+H02) 700, 700, 150
150 IF (X+H02) 151, 700, 700
151 ZMDNO = Z - DNO
TEST */ *Y + ZMDNO, ZMDNO
IF (TEST = RSRS) 152, 700, 700
152 JINDEX(1) = 1
NHIT(1) = NHIT(1) + 1
CALL ANGLE(X, Y, Z, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, TH) 

C CALL KINAM(A1T, A2T, 1.0087, QT, ET, TH, EN, EDUM)
IF (EN = ENTHG) 700, 700, 160
160 CALL DISTR(WT, TH, FT, NWT, 10)
DELTA = DELTA(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, XS, Z, RS, 1)
CALL INTRPL(MNT, ENT, SIGNT, EN, VSN)
ETAN = EXP(-VSN * DELTA)
FLUX = ETAN * SIN(TH) * VOL1

C CALL KINEV(TH, EN)

C CALL INTRPL(MG, EPA, SIGGP, EN, VSGP)
IF (HGP) 170, 170, 200
170 DSIGGP = VSGP / 4.0 / PI
G0 T0 203
200 D0 202 I1 = 1, MWGP
M = NWP(I)
D0 201 J1 = 1, M
A(J) = WGP(I, J)
201 B(J) = WGP(I, J)

202 CALL INTRPL(H, A, B, EN, W0RK(I))
CALL ANGLE(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, THNG)
CALL DISTR(WORK, THNG, SG, MWGP, 10)
DELTA = DELTA(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, XS, Z, RS, 1)
ETAG = EXP(-SIGGT * DELTG)
YLDG = FLUX * DSIGGP * ETAG / DG / DD

C UPDATE YLDSUM(1)

YLDSUM(1) = YLDSUM(1) + YLDG

C CHECK IF MULTIPLE SCATTERING CALCULATIONS ARE REQUESTED.
INITIALIZE PARAMETERS IF REQUIRED

IF (NSCAT = 1) 240, 700, 240
240 ISCAT = 2
XSAV1 = 0.0
YSAV1 = 0.0
ZSAV1 = 0.0
XSAV2 = X
YSAV2 = Y
ZSAV2 = Z
D0 250 I2 = NSCAT
J = 1
1
INDEX(J)=0

START OF MULTIPLE SCATTERING LOOP

SELECT SCATTERING POINT S(ISCAT)

R=RANF(1)
RR=VALUE(R,0,0,RMAX)
R=RANF(1)
TH=VALUE(R,0,PI)
R=RANF(1)
PHI=VALUE(R,0,2,0,PI)
SINTH=SIN(TH)
X=XSAV2+RR*SINTH*COS(PHI)
Y=YSAV2+RR*SINTH*SIN(PHI)
Z=ZSAV2+RR*COS(TH)
IF(X+HD2) 700,700,350
IF(X+HD2) 351,700,700
ZMDNO=Z-DNO
TEST=Y*Y+ZMDNO*ZMDNO
IF(TEST=RSRS) 352,700,700
JINDEX(ISCAT)=1
NHTT(ISCAT)=NHTT(ISCAT)+1
CALL ANGLE(X,Y,Z,XSAV2,YSAV2,ZSAV2,XSAV2,YSAV2,ZSAV2,XSAV1,YSAV1,THSCT)

SELECT NEUTRON SCATTERING PROCESS FOR POINT S(ISCAT=1)

D0 407 J=1,MNS
EB=DNSNS(I)*1.0*(1.0087/(A2NS(I)-1.0087))=(0.5*DNSNS(I))/(A2NS(I)-1.0)*GMSCT299
1087/1931.478)
IF(EB<EB) 400,400,401
400 DSNNS(I)=0.0
G0 TO 407
401 M=MNS(I)
D0 402 J=1,M
A(J)=ENS(I,J)
402 B(J)=SIGNS(I,J)
CALL INTRPL(M,A,B,EN,VSSCT)
IF(M(NNS(I))) 403,403,404
403 DSNNS(I)=VSSCT/4.0/P1
G0 TO 407
404 M=MNS(I)
D0 406 J=1,M
L=MNS(I,J)
D0 405 K=1,L
A(K)=EMSNS(I,J,K)
405 B(K)=EMSNS(I,J,K)
406 CALL INTRPL(L,A,B,EN,WORK(J))
CALL DISTRIBUT(WORK,THSCT,SSCT,M,10)
DSNNS(I)=VSSCT*SSCT/4.0/P1
407 CONTINUE
SUMSCT=0.0
D0 408 I=1,MNS
408 SUMSCT=SUMSCT+DSNNS(I)
IF(SUMSCT) 409,700,409
409 PSI(I)=0.0
K=MNS+1
PSI(K)=1.0
IF(MNS=1) 410,410,411
410 INDEX = 1
GOTO 414
411 SUMPSI = 0.0
DB 413 I = 2, MNS
J = 1
SUMPSI = SUMPSI + DSIGNS(J)
PSI(I) = SUMPSI / SUMSCT
IF (PSI(I) = 1.0) GOTO 413, 413, 412
412 PSI(I) = 1.0
413 CONTINUE
R = RANF(-1)
CALL FINDI (PSI, K, 6, R, INDEX)
414 NLEV(INDEX) = NLEV(INDEX) + 1
C
C CALCULATE NEUTRON ENERGY AND FLUX AT POINT S(ISCAT)
C
500 ENERGY = EN
CALL KINAM (1.0087, A2NS(INDEX), 1.0087, QNS(INDEX), EN, THSC, EN, FDUM)
IF (EN = ENTHG) 700, 700, 501
C
501 CALL INTRPL (MNT, ENT, SIGNT, EN, VSNT)
ETAN = EXF(-VSNT*RR)
FLUX = FLUX / SUMSCT * ETAN * SIN(TH) * V0LH
C
C CALCULATE GAMMA PRODUCTION FROM POINT S(ISCAT)
C
DG = SEPAR (X, Y, Z, 0.0, YD, ZD)
GAMMA = INTRPL (MG, EGP, SIGGP, EN, VSGP)
IF (MG) 502, 502, 600
C
502 SIGGP = VSGP / 4.0 / PI
GOTO 600
C
600 D = 602 I = 1, MWGP
M = MWGP(I)
D = 601 J = 1, M
A(J) = ENGP(I, J)
C
601 B(J) = WGP(I, J)
602 CALL INTRPL (MA, MB, EN, WORK(I))
CALL ANGLE(0.0, YD, ZD, X, Y, Z, XSAV2, YSAV2, ZSAV2, THNG)
CALL DISTR (WOKR, THNG, SG, MWGP, 10)
C
603 DELTG = DELTA(0.0, 0.0, DNO, 0.0, YD, ZD, X, Y, Z, RS = 0)
ETAG = EXF(-SIGGT * DELTG)
YLDG = FLUX * DG / SIGGP * ETAG / DG
C
C UPDATE YLDSSUM(ISCAT) AND FIX INDEX(ISCAT=1)
C
YLDSSUM(ISCAT) = YLDSSUM(ISCAT) + YLDG
J = ISCAT = 1
INDEX(J) = INDEX
C
C TEST FOR END OF MULTIPLE SCATTERING LOOP, RESET PARAMETERS FOR NEXT CYCLE IF REQUIRED
C
ISCAT = ISCAT + 1
IF (ISCAT = NISCAT) 610, 610, 700
C
610 XSAV1 = XSAV2
YSAV1 = YSAV2
ZSAV1 = ZSAV2
XS = XS
YS = YS
ZS = ZS
C
-39-
G0 TO 300

C XXX END OF MULTIPLE SCATTERING LOOP

C UPDATE HISTORIES AVERAGE, INTERMEDIATE OUTPUT IF SS1 UP, TEST FOR COMPLETION OF REQUESTED NUMBER OF HISTORIES OR SS2 UP AND PROCEED TO OUTPUT OR CONTINUE WITH HISTORY ACCORDING TO OUTCOME

700 DO 701 I=1,NSCAT
701 YLD(I)=YLDSUM(I)/FLOAT(IHIST)
   CALL SSWITCH(1,K1)
   IF(K1=2) 702,703,702
702 WRITE(5,703) IHIST
703 FORMAT(517)
   WRITE(5,703) (INDEX(J),J=1,NSCAT)
   IF(NSCAT=1) 704,705,704
704 I=NSCAT+1
   WRITE(5,703) (INDEX(J),J=1,I)
   WRITE(5,703) (NLEV(J),J=1,MNS)
705 WRITE(5,703) (NHIT(J),J=1,NSCAT)
   WRITE(5,22) (YLD(I),I=1,NSCAT)
706 CALL SSWITCH(2,K2)
   IF(K2=1) 707,800,707
707 IF(IHIST=NHIST) 708,800,800
708 IHIST=IHIST+1
   G0 TO 100
C C END OF HISTORY LOOP
C C
C FINAL OUTPUT=IF NSCAT,GT,1 RENORMALIZE YLD(I) SO YLD(I)=1 AND
C COMPUTE ALFA

800 WRITE(5,801) IHIST
801 FORMAT(10WHISTORIES=,18)
   IF(NSCAT=1) 804,804,802
802 WRITE(5,803)
803 FORMAT(7HNLEV(I))
   WRITE(5,703) (NLEV(I),I=1,MNS)
804 WRITE(5,805)
805 FORMAT(7HNHT(I))
   WRITE(5,703) (NHIT(I),I=1,NSCAT)
   WRITE(5,806)
806 FORMAT(8HYLD(I))
   WRITE(5,22) (YLD(I),I=1,NSCAT)
   IF(NSCAT=1) 807,1,807
807 FYLD=YLD(1)
   D0 808 I=1,NSCAT
808 YLD(I)=YLD(I)/FYLD
   ALFA=0.0
   D0 809 I=2,NSCAT
809 ALFA=ALFA+YLD(I)
   WRITE(5,810)
810 FORMAT(17HNORMALIZED YLD(I))
   WRITE(5,22) (YLD(I),I=1,NSCAT)
   WRITE(5,811) ALFA
811 FORMAT(5HALFA=,E10.4)

C IF(K2=1) 1,812,1
812 PAUSE
   G0 TO 1
FUNCTION DELTA(X0,Y0,Z0,X1,Y1,Z1,X2,Y2,Z2,RS,INDEX)

FUNCTION TO DETERMINE PENETRATION DEPTH FOR R.C. CYLINDER.
INDEX = 1, 2 OR 3 IMPLIES CYLINDER AXIS PARALLEL TO X, Y OR Z-AXIS
RESPECTIVELY. (X0,Y0,Z0) IS CYLINDER CENTER, (X1,Y1,Z1) IS EXTERIOR POINT, (X2,Y2,Z2) IS INTERIOR POINT. RS = CYLINDER RADIUS

R = SQRT(((X2-X1)^2+(Y2-Y1)^2+(Z2-Z1)^2))
A = (X2-X1)/R
B = (Y2-Y1)/R
C = (Z2-Z1)/R
G0 T0 0, 1, 2, 3, INDEX
1 Y0W = Y0
Z0W = Z0
Y1W = Y1
Z1W = Z1
B0H = B
C0W = C
G0 T0 4
2 Y0H = Y0
Z0W = Z0
Y1H = Y1
Z1H = Z1
B0C = B
C0A = C
G0 T0 4
3 Y0W = Y0
Z0W = Z0
Y1W = Y1
Z1W = Y1
B0H = A
C0W = B
G0 T0 6
4 S = (B0W*(Y0W-Y1W)+C0W*(Z0W-Z1W)+SQRT((B0W*B0W+C0W*C0W)*RS*RS-(C0W*(Y1W-Y0W)-B0W*(Z1W-Z0W))))/(B0W*B0W+C0W*C0W)
DELTA = S
RETURN
END

SUBROUTINE KINAM(A1,A2,A3,O,E1,TH3,E31,E32)

W1 = 931.478*A1
W2 = 931.478*A2
W3 = 931.478*A3
W4 = W1+W2+W3=0
EF = O*(1.0*(W1/W2)=(0.5*(W2/2))
EB = O*(1.0*(W1/(W2=W3))=(0.5*(W2=W3))
IF(E1=EF) 1, 1, 2
1 E31=0.0
11 E32=0.0
G0 T0 6
2 C = COS(TH3)
A = 2.0*(W3+W4+E1*O)
B = 2.0*E1*(W1=W4)/(2.0*W4*O)*O)
D = E1*(E1+2.0*W1)*C*C
TERM = (B0B*2.0-W3*A*B+4.0*W3*W3*O)*E1*(E1+2.0*W1)
IF(TERM) 1, 1, 3
3 DEN = A*A=4.0*D
U = (4.0*W3*D*A*B)/DEN
V = 2.0*C*SQRT(ABS(TERM))/DEN
E31=U*V
IF(E1=0) 4,4.5
4 IF(TH<1.5707963) 41,11,11
41 E32=U*V
G0 T0 6
5 E32=E31
6 RETURN
END
SUBROUTINE ANGLE(X1H,Y1H,Z1H,X1T,Y1T,Z1T,X2H,Y2H,Z2H,X2T,Y2T,Z2T,TG)
V1=SORT(ABS((X1H-X1T)*(Y1H-Y1T)*(Z1H-Z1T)*(Z1H-Z1T)*1
V2=SORT(ABS((X2H-X2T)*(Y2H-Y2T)*(Z2H-Z2T)*(Z2H-Z2T)*1
1-2T))
D0T=(X1H-X1T)*(X2H-X2T)+(Y1H-Y1T)*(Y2H-Y2T)+(Z1H-Z1T)+(Z2H-Z2T)
TH=ARCCOS(CH+2)
RETURN
END
SUBROUTINE DISTRIW(V,T,H,NMAX)
DIMENSION W(NMAX)
V=1,0
IF(NW.EQ.0) G0 T0 4
D0 2 I=1,NW
IF(H(I)) 21,20,21
20 VADD=0,0
G0 T0 22
21 VADD=W(I)*POLYL(2,1,TH)
22 V=V+VADD
2 CONTINUE
IF(V) 3,4,4
3 V=0,0
4 RETURN
END
FUNCTION POLYLI(IOP,N,ANGLE)
X = ANGLE
G0 TO(10,11,12,10P)
10 X = .017453293*X
11 X = COS(X)
12 NBIG = N-1
IF(NBIG) 1,2,3
1 POLYL = 1,0
G0 TO 100
2 POLYL = X
G0 TO 100
3 POLYL = X
G0 TO 100
PLM1 = 1,0
D0 4 L=1,NBIG
POLYL = (FLOAT(2*L+1)*X*PL = FLOAT(L)*PLM1)/FLOAT(L+1)
PLM1 = PL
4 PL = POLYL
100 RETURN
END
SUBROUTINE INTRPL(N,X,T,Y,L,Y)
DIMENSION X(N),Y(N)
IF(X<T(1)) 1,3,4
1 WRITE(1,2)
2 FORMAT(8HRANG ERR)
PAUSE
3 Y=YT(1)
4 IF(X=XT(N)) 7,5,1
5 Y=YT(N)
6 G0 T0 24
7 I=0
8 J=N
9 K=0.5*FLOAT(J-1)+0.1
10 IF(X=XT(K)) 9,10,11
11 J=K
12 I=K
13 IF(J=I=1) 13,13,8
14 I=J
15 J=I+1
16 DEN=XT(J)-XT(I)
17 C1=(XT(J)*YT(I)-XT(I)*YT(J))/DEN
18 C2=(YT(J)-YT(I))/DEN
19 Y=C1+C2*X
20 RETURN
21 END
22 FUNCTION ARCCOS(X,K)
23 ARCCOS=1.5707963
24 IF(ABS(X),GT,0.999999) X=.999999*X/ABS(X)
25 IF(X*X,LT,1.0E-70) ARCCOS=ATAN(SQRT(ABS(1./X/X-1.)))
26 IF(X,LT,0.) ARCCOS=3.1415926-ARCCOS
27 G0 T0 (100,200),K
28 RETURN
29 ARCCOS=ARCCOS+57.2957795
30 RETURN
31 END
32 FUNCTION EXP(Z)
33 IF(Z),1,3
34 1 IF(Z,LT,-70.0) Z=-70.0
35 IF(Z,LT,-1E+04) G0 T0 2
36 EXP=EXP(Z)
37 G0 T0 4
38 3 IF(Z,GT,70.0) Z=70.0
39 IF(Z,LT,1E+04) G0 T0 2
40 EXP=EXP(Z)
41 G0 T0 4
42 2 EXP=1.0*Z
43 CONTINUE
44 RETURN
45 END
46 SUBROUTINE FINDI(Y,N,NDIM,Z,IZ)
47 DIMENSION Y(NDIM)
48 NMIN=1
49 NMAX=N
50 INTER=0.5*FLOAT(NMAX-NMIN)+0.1
51 NTEST=NMIN+INTER
52 IF(Z,Y(NTEST)) 1,2,3
53 1 NMAX=NTEST
54 G0 T0 4
55 2 IZ=NTEST
56 G0 T0 999
57 3 NMIN=NTEST
58 4 IF(NMAX=NMIN=1) 5,5,36
59 5 IZ=NMAX=1
-43-
REFERENCES

14. Evaluated Neutron Data File, ENDF/B-IV, National Neutron Cross Section Center, Brookhaven National Laboratory.
Table I

Nominal Conditions Considered in Sample Geometry and Absorption Calculations

Geometry:
\[ R_s = 1.9 \text{ cm}, \ H = 3.8 \text{ cm}, \ D_n = 12.8 \text{ cm}, \]
\[ D_\gamma = 130.0 \text{ cm}, \ a = 0.5 \text{ cm}, \ \text{various } \theta_{\text{DET}} \]

Mesh size:
\[ n_T = 1, \ n_H = 5, \ n_R = 5, \ n_\phi = 3 \]

Sample material and gamma ray:
Natural iron, \( E_\gamma = 0.846 \text{ MeV} \)

Neutron source:
Isotropic yield, \( Q = 0, \ A_1 = 1, \ A_2 = 7. \)
Incident energy selected so \( E_{n,\text{max}} = 1 \text{ MeV}. \)

Cross sections\(^a\):
\[ \Sigma_{nT} = 0.27, \ \Sigma_{\gamma T} = 0.5 \]
\[ (d\sigma/d\Omega)_\gamma = 0.038 \text{ b/sr (isotropic)} \]

Gamma-detector efficiency:
Arbitrary constant value

\(^a\)These are nominal values which are not necessarily equal to ENDF/B-IV values for iron.
<table>
<thead>
<tr>
<th>Description</th>
<th>&quot;Front&quot; - to &quot;back&quot; ratio(^a)</th>
<th>&quot;Near&quot; - to &quot;far&quot; ratio(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>i) Neutron and gamma-ray absorption</td>
<td>2.11</td>
<td>2.11</td>
</tr>
<tr>
<td>ii) Gamma-ray absorption only</td>
<td>1.19</td>
<td>2.19</td>
</tr>
<tr>
<td>iii) Neutron absorption only</td>
<td>1.78</td>
<td>1.03</td>
</tr>
<tr>
<td>iv) No absorption</td>
<td>1.16</td>
<td>1.02</td>
</tr>
</tbody>
</table>

\(^a\) Ratios are defined in Section 3.2.
Table III

Dependence of Gamma-Ray Yield on the Sample Radius

<table>
<thead>
<tr>
<th>$R_S$ (cm)</th>
<th>$\bar{Y}_0$</th>
<th>$R_S \bar{Y}_0$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.635</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0.95</td>
<td>0.84</td>
<td>1.25</td>
</tr>
<tr>
<td>1.27</td>
<td>0.70</td>
<td>1.41</td>
</tr>
<tr>
<td>1.59</td>
<td>0.60</td>
<td>1.50</td>
</tr>
<tr>
<td>1.905</td>
<td>0.52</td>
<td>1.55</td>
</tr>
<tr>
<td>2.54</td>
<td>0.39</td>
<td>1.57</td>
</tr>
</tbody>
</table>

$^{a}\bar{Y}_0$ computed for $\theta_{DET} = 90^\circ$. Values are relative to corresponding values for $R_S = 0.635$ cm.
Table IV

Nominal Conditions Considered in
Sample Multiple Scattering Calculations

Geometry:
\[ R_s = 1.9 \text{ cm}, \quad H = 3.8 \text{ cm}, \quad D_n = 11.4 \text{ cm}, \]
\[ D_\gamma = 130.0 \text{ cm}, \quad \text{Various } \theta_{DET} \]

Sample material and gamma ray:
Natural iron, \( E_\gamma = 0.846 \text{ MeV} \)

Neutron source:
Isotropic yield, \( Q = 0, \ A_1 = 1, \ A_2 \) = very large.
Neutron energy \( E_n = 2 \text{ MeV} \).

Cross sections\(^a\):
\[ E_nT = 0.17 \]
Elastic scattering, \( Q = 0, \ A_1 = 1, \ A_2 = 55.85, \)
\[ E_{EL} = 0.17 \text{ (isotropic)} \]
Inelastic scattering, \( Q = -0.85 \text{ MeV}, \ A_1 = 1, \ A_2 = 55.85, \)
\[ E_{IN} = 0.05 \]
\[ (d\sigma/d\Omega)_\gamma = 0.038 \text{ b/sr (isotropic)} \]

\(^a\) These are nominal values which are not necessarily equal to ENDF/B-IV values for iron.
Table V

Effect of Coherent Photon Scattering

<table>
<thead>
<tr>
<th>Sample Material</th>
<th>$E_\gamma$ (MeV)</th>
<th>Ratio $Y_U/(Y_U + Y_S)^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li (Z=3)</td>
<td>0.1</td>
<td>1.00</td>
</tr>
<tr>
<td>Al (Z=13)</td>
<td>0.1 0.3 0.5</td>
<td>0.96 0.99 1.00</td>
</tr>
<tr>
<td>Ti (Z=22)</td>
<td>0.1 0.3 0.5 0.8</td>
<td>0.95 0.98 0.99 1.00</td>
</tr>
<tr>
<td>Fe (Z=26)</td>
<td>0.1 0.3 0.5 0.8</td>
<td>0.98 0.98 0.99 1.00</td>
</tr>
<tr>
<td>Zn (Z=30)</td>
<td>0.1 0.3 0.5 0.8</td>
<td>0.99 0.97 0.99 0.99</td>
</tr>
<tr>
<td>Mo (Z=42)</td>
<td>0.1 0.3 0.5 0.8</td>
<td>1.00 0.97 0.98 0.99</td>
</tr>
</tbody>
</table>

$^a$ $Y_U$ and $Y_S$ are computed using Eqn. (81) and (82). Values of the ratio for larger $E_\gamma$ are $\sim 1.00$ if not given in the table.
FIGURE CAPTIONS

Fig. 1. The total cross section for natural iron in the energy range 0.1-2 MeV. The solid curve represents ENDF/B-IV values [14]. The dashed curve represents the same information after smoothing with a 0.1-MeV resolution function. The smoothed excitation function can be approximated by connecting the large dots with straight line segments. This simulates linear interpolation of a lookup table which is stored in the memory of a digital computer. (ANL Neg. No. 116-75-91).

Fig. 2. Schematic diagrams to illustrate geometry applicable to computation of the yield of gamma-rays from (n,γ) reactions produced by unscattered neutrons. (ANL Neg. No. 116-75-89).

Fig. 3. Distortion of an isotropic gamma-ray production angular distribution by radiation absorption. (ANL Neg. No. 116-75-85).

Figs. 4 thru Fig. 7 Sample midplane relative-yield profiles for the following respective conditions: i) neutron and gamma-ray absorption, ii) gamma-ray absorption only, iii) neutron absorption only, and iv) no absorption. (ANL Neg. Nos. 116-75-84, 116-75-88, 116-75-86, 116-75-82).

Fig. 8. Computed angular resolution functions for several values of θ_{DET} and Table I parameters. (ANL Neg. Nos. 116-75-80).

Fig. 9. Demonstration of the factorization rule. The solid lines represent assumed differential cross section functions while the solid circles represent values of Υ_0/(1 - Δcos θ_{DET}) for various θ_{DET} but plotted at the corresponding angles <θ_HY>. All results are normalized to unity at θ_{DET} = 90°. (ANL Neg. No. 116-75-87).

Fig. 10. Geometry appropriate to multiple-scattering calculations for the first two scattering orders. (ANL Neg. No. 116-75-92).

Fig. 11. Plot of relative values for Y_0, Y_1, Y_2 and Y_3 computed using the parameters in Table IV. (ANL Neg. No. 116-75-90).

Fig. 12. Plots of α_1, α_2, α_3 and α_{TOT} for various sample sizes (H = 2 R_s). (ANL Neg. No. 116-75-83).

Fig. 13. The relative yield of 0.846-MeV gamma rays per atom for various natural iron samples. Comparison is made between the experimental results and the results of four sets of computations described in Section 5. (ANL Neg. No. 166-75-81).

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NEUTRON TOTAL CROSS SECTION FOR IRON

- ENDF/B-IV
- SMOOTHED (0.1 MEV RESOLUTION)
- INTERPOLATION TABLE
ALL YIELD CURVES NORMALIZED TO UNITY AT 90 DEGREES

GAMMA-RAY YIELD, RELATIVE

θ_DET, DEGREES

- GEOMETRY
- GEOMETRY + NEUTRON ABSORPTION
- GEOMETRY + GAMMA-RAY ABSORPTION
- GEOMETRY + NEUTRON AND GAMMA-RAY ABSORPTION
Fig. 5

GAMMA-RAY ABSORPTION

GAMMA-RAY DETECTOR

INCIDENT NEUTRONS
Fig. 6

NEUTRON ABSORPTION

GAMMA-RAY DETECTOR

INCIDENT NEUTRONS
Fig. 8

\[ \theta_{\text{DET}} = 0^\circ \]
\[ \langle \theta \rangle = 6.8^\circ \]
\[ \text{RESOL} = 6^\circ \]

\[ \theta_{\text{DET}} = 30^\circ \]
\[ \langle \theta \rangle = 29.2^\circ \]
\[ \text{RESOL} = 12^\circ \]

\[ \theta_{\text{DET}} = 60^\circ \]
\[ \langle \theta \rangle = 58.1^\circ \]
\[ \text{RESOL} = 12^\circ \]

\[ \theta_{\text{DET}} = 90^\circ \]
\[ \langle \theta \rangle = 87.9^\circ \]
\[ \text{RESOL} = 11^\circ \]

\[ \theta_{\text{DET}} = 120^\circ \]
\[ \langle \theta \rangle = 118.3^\circ \]
\[ \text{RESOL} = 12^\circ \]

\[ \theta_{\text{DET}} = 150^\circ \]
\[ \langle \theta \rangle = 148.8^\circ \]
\[ \text{RESOL} = 13^\circ \]

\[ \theta_{\text{DET}} = 180^\circ \]
\[ \langle \theta \rangle = 174.2^\circ \]
\[ \text{RESOL} = 5^\circ \]

\[ \theta = \text{COMPUTED VALUES OF THE ANGULAR RESOLUTION FUNCTIONS} A(\theta) \text{ AT ONE-DEGREE INTERVALS FOR VARIOUS } \theta_{\text{DET}} \]
\[ \langle \theta \rangle = \text{AVERAGE SCATTERING ANGLE COMPUTED USING } A(\theta) \]
\[ \text{RESOL} = \text{APPROXIMATE FWHM OF THE DISTRIBUTION } A(\theta) \]

SCATTERING ANGLE, DEGREES

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ANGULAR DISTRIBUTIONS NORMALIZED TO UNITY AT 90 DEGREES

\[ \frac{Y_0(\theta)}{(1 - \Delta \cos \theta)} \text{ PLOTTED AT } \langle \theta \rangle \]

MOMENTUM TRANSFER DISTRIBUTION

\[ \langle d\sigma/d\Omega \rangle = 1 + W_2 P_2(\cos \theta) \]

\( W_2 = -0.2 \)

\( W_2 = -0.5 \)
Fig. 10

\( Y_0 \) - Unscattered Neutrons

Neutron Source (Origin)

\( Y_1 \) - Once-Scattered Neutrons

Neutron Source (Origin)

---

Neutron Path

Gamma-Ray Path
Fig. 11

- MONTE-CARLO CALCULATIONS
- \((1 - \Delta \cos \theta_{\text{DET}})\) FITS

\(Y_0\) - UNSCATTERTED NEUTRONS

\(Y_1\) - SINGLY-SCATTERED NEUTRONS

\(Y_2\) - DOUBLY-SCATTERED NEUTRONS

\(Y_3\) - TRIPLY-SCATTERED NEUTRONS

\(\theta_{\text{DET}}, \text{ DEGREES}\)

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Fig. 12

\[ \alpha_{\text{TOT}} = \alpha_1 + \alpha_2 + \alpha_3 \]